On Greening Cellular Networks by Sharing Base Stations: A Game-theoretic Approach *

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ABSTRACT

With increasing demands for mobile data traffic and efforts for a better QoS, many base stations (BSs) consume a significant amount of electrical power with a lot of electricity bill. Many practical solutions include sharing BSs among mobile network operators (MNOs), in which an MNO's BS allows to serve traffic from nearby users subscribing to other MNOs with being paid a certain roaming fee. However, without assurance of gains, MNOs would not agree to BS sharing. In this paper, we study pricing and user association policies that assure actual gains to each MNO in the BS sharing. We model this with a game that jointly involves the strategic decision of roaming price and user association, where we consider the flow-level dynamics of traffic. We assume a time-scale separation where pricing decision is made at a slower time scale than user association, as often done in practice. First, for a fixed roaming price we analyze the user association game, where we prove that (i) it is a potential game, (ii) there exists a unique pure Nash equilibrium (NE), and (iii) a distributed algorithm inspired by an approximate version of Jacobi play converges to the NE. Based on this nice properties of the user association at a faster time scale, we study the slower time-scale pricing decision game and prove that there exists a pure NE with achieving almost the efficiency of full-cooperation (without roaming fee). We demonstrate that there exists a significant degree of energy saving, once an appropriate competition rule is provided, through numerical simulations under a variety of scenarios including those based on a real 3G deployment.

Categories and Subject Descriptors

C.2.0 [**General**]: Data communications; C.2.1 [**Network Architecture and Design**]: Wireless communication; C.2.3 [**Network Operations**]: Network management

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Keywords

Greening, User association, BS sharing, Game theory

1. INTRODUCTION

Demands for data service in cellular networks are highly increasing, as predicted by Cisco [1], where mobile data traffic will reach about 24 Exabytes per month by 2019. To cover increasing demands for data traffic, each MNO improves spectral efficiency by densely deploying small-cell BSs. However, densely-deployed BSs naturally require high expense in power consumption. Multiple MNOs in each country (three major MNOs in Korea, for example), which experience high market competition, independently deploy their network in most cases, and thus it is reported that there are significantly redundant overlaps (but with different geographical locations) in the coverage of BSs among multiple MNOs, as exemplified in the BS deployment map provided by UK government-sponsored website [2] (see Figure 1). A notion of BS sharing can be considered as a candidate solution to significantly reduce the power consumption while maintaining user-level QoS. As the name implies, the key idea of BS sharing is to break the boundary of BS ownership, allowing users to be served by nearby BSs, regardless of their original subscription by paying a certain roaming fee. However, without an appropriate choice of roaming price and a rule of associating users with suitable BSs, the effect of BS sharing would not be maximized. High competition among existing MNOs requires to study these under a strategic situation, where each MNO selfishly determines its own roaming fee as well as user association (i.e., which users would be serviced by which BSs).

Main contribution. In this paper, we study the policies of charging roaming fee and associating users under a certain roaming agreement, where each MNO strategically tries to minimize their operating expenditure (OPEX). The roaming pricing policy determines how much each MNO charges "outside" users for their roamed traffic, and the user association policy determines the actual amount of roamed traffic, both of which depend on a few key factors, such as the number and the location of BSs and subscribers in each MNO. Clearly, these two policies jointly determine the payoff of an MNO. For instance, the cheaper roaming fee of an MNO would increase the amount of roamed traffic, where separately assigning the roamed traffic to the BSs of other MNOs would differently determine its OPEX. The main contribution of this paper is summarized as follows:

- Having pricing and user association in one game leads to a highly challenging mathematical model, often losing tractability. More importantly, as is usual in practice, users are often reluctant to accept fast-changing prices. Thus, we take a time-separation ap-

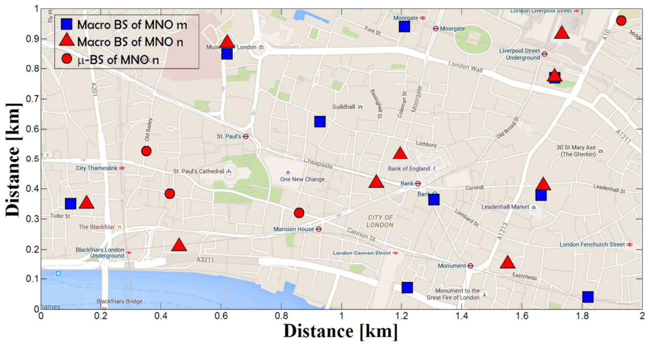


Figure 1: Real 3G deployment map in London, UK (9 BSs of MNO m and 13 BSs of MNO n in 2 x 1 [km^2])

proach that those two strategic factors affect the system at different time scales. We assume that the pricing decision game is played at a slower time scale than the user association game. Another key modeling feature of this paper lies in our perspective of QoS, where we consider a flow-level performance (such as file-transfer delay), as done in [3], that seems to provide more realistic QoS measure to the users, compared to other typical approaches that are based on packet-level throughput.

- In the analysis of game, first, for a fixed roaming price vector of MNOs, in the user association game we study how each MNO splits the traffic of its subscribers across the entire BSs in the network. The challenges lie in (a) the complex inter-play of flow-level user performance and BS power consumptions, which depend on the amount of roamed traffic from other MNOs as well as (b) finding a distributed user association mechanism. We first show that this game is a potential game, and thus the useful features such as existence/uniqueness of NE are provably ensured. Next, we propose a low-complexity, distributed dynamic update algorithm that provably converges to the NE, inspired by an approximate version of *Jacobi play*. These nice properties of the user association game in terms of equilibrium and dynamics play a role of providing a good underlying module to the pricing game.
- Using the underlying user association game running at a faster time scale, we study the pricing decision game, where each MNO strategically chooses its roaming price. We show the existence of a pure NE using quasi-convexity of the payoff function, and we also analyze various aspects of the NE through extensive simulations. Based on our analysis of pricing decision and user association games, we draw useful engineering implications on how MNOs decide on their roaming price and how much energy saving is achieved under BS sharing whose key factors are strategically determined. Interesting messages include the one that at equilibrium, the equilibrium almost achieves the efficiency of full-cooperation under various environments (0.96-0.99), showing much better efficiency than that in conventional non-BS sharing cases (0.65-0.88).

Related work. The idea of BS sharing is conceptually suggested in [4,5]. They proposed a BS sharing scheme that the remaining traffic of some BSs is accepted by different operators as roaming traffic, when some BSs are switched off. However, it is not guaranteed that MNOs have sufficient incentive to perform BS sharing, since they do not consider the economic gain under a roaming agreements between MNOs. There are several works to analyze economic incentives of BS sharing under roaming agreements [6–11]. The authors in [6] studied the possibility that a roaming agreement may lead to a collusion between MNOs. In [7], the authors studied the trade-off between BS sharing and investments for cellular network capacity using the newsvendor model. In [8], the authors considered only electrical bill and roaming fee in their proposed algorithm for BS sharing irrespective of user QoS. In [9], the authors proposed a user association algorithm in sharing BSs for uplink throughput maximization using the approach of Nash bargaining to share revenue from the BS sharing. The authors in [10, 11] analyzed the economic benefits from BS switching on/off with BS sharing. They focus on the trade-off between throughput and energy saving in cellular networks, where for simplicity it was assumed that multiple MNOs have the identical BS deployment map. Such an assumption makes the user association problem simple due to the coexistence of BSs of other MNOs, resulting in a user association algorithm that each BS always serves the other MNOs' traffic when the BS of other MNOs in the same location was switched off. However, in real BS deployment, BSs are often located in a heterogeneous manner, as shown in Fig. 1. User association schemes for a single MNO are proposed in [3, 12–19] under different models and assumptions, where most works consider the received signal strength and the amount of traffics in BSs to select a serving BS without considering power consumption. The recent studies [3, 17–20] considered flow-level performance in associating users in consideration of power consumption, some of which motivates our user association game in the context of multiple MNOs.

2. MODEL

2.1 System Model

Network and BS sharing service. We consider a wireless cellular network with a set $\mathcal{M}$ of multiple MNOs. For simplicity, we use $-m \doteq \mathcal{M} \setminus \{m\}$. Denote by $\mathcal{B}_m$ the set of BSs owned and served by operator m, where we let the entire set of BSs be $\mathcal{B}$, i.e., $\mathcal{B} = \bigcup_{m \in \mathcal{M}} \mathcal{B}_m$. We abuse the notation of m, where we use $m(b)$ to indicate the MNO that owns the BS b. We assume that the BS ownership is exclusive and all users subscribe to only one MNO. We consider the existence of BS sharing that any user can be associated with a BS regardless of her original subscription, as done in [4–11]. However, when she is associated with and served by a BS owned by other MNOs than her MNO, she or her MNO has to pay a certain fee called *roaming fee* which we will discuss later. We also assume that all BSs do not differentiate in service priority between traffics from their subscribers and roamed traffics.

Traffic and capacity. We consider a region $\mathcal{L} \subset \mathcal{R}^2$ that is covered by the BSs of all MNOs. We assume that each flow in MNO m arrive as a location-dependent inhomogeneous Poisson point process with rate $\lambda_m(x)$ at a location $x \in \mathcal{L}$, and the file size of each arrival is independently distributed with mean $1/\mu_m(x)$. Thus, the traffic intensity at location x for MNO m is $\gamma_m(x) \doteq \frac{\lambda_m(x)}{\mu_m(x)} < \infty$. We consider that each user at a location x experiences the same data rate $c_x^b = c_x^b(\mathcal{B}_{m(b)})$ for a given BS b associating the user (i.e., the same channel conditions at the same location). Note that the dependence on $\mathcal{B}_{m(b)}$ is due to the interference among the BSs of an MNO, but the capacity does not depend on the BSs of other MNOs, since each MNO operates the network in a difference frequency band. We further assume that the data rate is not changed over time, i.e., we do not consider fast fading or dynamic inter-cell interference, since the time scale of fast-fading and inter-cell interference is much faster than the time scale of user association and pricing decision. Hence the fast-fading and inter-cell interference are considered as Gausssian-like noise in user association [3, 13, 15, 17].

Loads. For a given BS b, we define *system-load density* of location x to be $\varrho_m^b(x) \doteq \frac{\gamma_m(x)}{c_x^b}$, which denotes the fraction of time required to deliver traffic intensity $\gamma_m(x)$ from the BS b to location x. We introduce the notion of association vector $\boldsymbol{y}_m^b \doteq (y_m^b(x) : x \in \mathcal{L})$, where for any given MNO m and a BS b, $y_m^b(x) \in [0,1]$ corresponds to the fraction of time that the users of MNO m at location x are associated with BS b, thus, we should have $\sum_{b\in\mathcal{B}} y_m^b(x) = 1$. For notational convenience later, for a given BS b, we denote $\boldsymbol{y}^b \doteq (\boldsymbol{y}_m^b : m \in \mathcal{M})$, and also denote the entire association $\boldsymbol{y} \doteq (\boldsymbol{y}^b : b \in \mathcal{B})$. For a given association vector $\boldsymbol{y}$, we are interested in the offered load imposed on a BS b of MNO m, which we denote by $\rho_m^b = \rho_m^b(\boldsymbol{y})$,[1] because the offered load affects the QoS, the consumed power, and the roaming fee, all of which will be directly related to the aggregate utility of MNOs (see Section 3), i.e.,

$$\rho_m^b = \int_{\mathcal{L}} \varrho_m^b(x) \cdot y_m^b(x)\, dx. \tag{1}$$

Note that if b is the BS that the MNO m does not own, (i.e., $b \in \mathcal{B}_{-m}$), ρ_m^b is the load of the *roamed traffic.* Then, for a given $\boldsymbol{y}$, we can define the *system load* by MNO m by a vector $\boldsymbol{\rho}_m \doteq (\rho_m^b : b \in \mathcal{B})$, which corresponds to the collection of the loads across BSs assigned by MNO m. Similarly, we denote per-BS aggregate load (offered by all MNOs) by $\rho^b = \sum_{m\in\mathcal{M}} \rho_m^b$, and finally let $\boldsymbol{\rho} = (\rho^b : b \in \mathcal{B})$. Then, the set $\mathcal{F}$ of feasible loads that corresponds to the collection of loads achieved by all possible association vectors under the stability of each BS, is given by:

$$\begin{aligned}\mathcal{F} = \{\boldsymbol{\rho} \quad | \quad & \rho^b(\boldsymbol{y}) \le 1-\epsilon,\ \forall b \in \mathcal{B},\\ & \sum_{b\in B} y_m^b(x) = 1,\ \forall m \in \mathcal{M}, \forall x \in \mathcal{L},\\ & y_m^b(x) \in [0,1], \forall m \in \mathcal{M},\ \forall x \in \mathcal{L}, \forall b \in \mathcal{B}\},\end{aligned}$$

where ϵ is an arbitrary small positive constant.

3. PROBLEM FORMULATION: GAME

We consider the case where MNOs are strategic and selfishly maximize their revenue (or equivalently minimize their cost). To that end, we formulate a game played by the MNOs, where their strategies of an MNO, say m, are: first, $\boldsymbol{\rho}_m$, how much traffic to handle in which BSs (by controlling the association vector), and second, k_m, the roaming price, i.e., the amount of money that m charges for the roamed traffic from $-m$. Note that MNOs control the association vectors $\boldsymbol{y}$ rather than $\boldsymbol{\rho}$, but due to the relationship between $\boldsymbol{y}$ and $\boldsymbol{\rho}$ in the previous section, we henceforth consider $\boldsymbol{\rho} \in \mathcal{F}$ as a strategy vector.

Formally, we model the payoff function of MNO m corresponding to the cost function that should be minimized:

$$\begin{aligned}\mathcal{U}_m(\boldsymbol{\rho}_m, \boldsymbol{\rho}_{-m}, k_m, k_{-m}) \doteq & \underbrace{\sum_{b\in\mathcal{B}} \phi_\alpha(\rho^b)}_{(a)} + \eta \underbrace{\sum_{b\in\mathcal{B}_m} \mathcal{E}^b(\rho^b)}_{(b)}\\ & \underbrace{- \sum_{b\in\mathcal{B}_m}\sum_{n\in-m} \eta\Delta\mathcal{E}^b(\rho_n^b) - k_m \sum_{b\in\mathcal{B}_m}\sum_{n\in-m} g^b(\rho_n^b)}_{(c)}\\ & \underbrace{+ \sum_{b\in\mathcal{B}_{-m}} \eta\Delta\mathcal{E}^b(\rho_m^b) + \sum_{b\in\mathcal{B}_{-m}} k_{m(b)} g^b(\rho_m^b)}_{(d)}, \end{aligned} \tag{2}$$

where each function and parameter are explained shortly, and each term is interpreted as: *(a)* is the cost of experienced QoS[2], *(b)* is the power operating cost of MNO m owns, *(c)* is the revenue from serving other MNO's roamed traffic, and *(d)* is the cost to pay for MNO m's roamed traffic to other MNOs.

(i) QoS cost $\phi_\alpha(\cdot)$***:*** The function $\phi_\alpha(\cdot)$ represents the flow-level QoS cost function (such as file-transfer delay), modeled by:

$$\phi_\alpha(\rho^b) = \begin{cases} \frac{(1-\rho^b)^{1-\alpha}}{\alpha-1}, & \text{if } \alpha \neq 1,\\ \log(\frac{1}{1-\rho^b}), & \text{if } \alpha = 1, \end{cases} \tag{3}$$

where $\alpha \ge 0$ is a parameter that characterizes the incurred QoS cost. The case $\alpha = 0$ (often called rate-optimal) corresponds to the cost measured by user rates, since $\phi_0(\rho^b)$ becomes $\sum \rho^b$, and then, a user gives the work load to a BS who has maximum capacity. When $\alpha = 2$, the function represents that the summation of the average number of users (similarly delay from Little's formula) across all BSs from the standard queueing theory. This function is first used in [3].

(ii) BS power cost $\mathcal{E}^b(\cdot)$***:*** The function $\mathcal{E}^b(\cdot)$ represents the amount of power consumed in BS b, consisting of (i) offset power for turning on a BS and (ii) adaptive power that increases as the BS utilization, modeled by: for each BS b,

$$\mathcal{E}^b(\rho^b) = \beta^b E^b \rho^b + (1-\beta^b)E^b, \tag{4}$$

where $\beta^b \in [0,1]$ quantifies the portion of the load proportional power consumption, and E^b is the maximum operational power when fully utilized (i.e., $\rho^b = 1$). The case when $\beta^b = 0$ means that BS b is ideally *energy-proportional,* but typically, the range of β^b is roughly $0.5 - 0.8$ in real UMTS BS [21].

(iii) Roaming fee $\eta\Delta\mathcal{E}^b(\cdot)$ ***and*** $g^b(\cdot)$***:*** The roaming fee consists of two parts: (i) *prime cost (*$\eta\Delta\mathcal{E}^b(\cdot)$*),* and (ii) *profit margin (*$g^b(\cdot)$*),* where

$$\Delta\mathcal{E}^b(x) \doteq \beta^b E^b x, \quad g^b(x) \doteq \eta\Delta\mathcal{E}^b(x) + x. \tag{5}$$

The function $\Delta\mathcal{E}^b(x)$ quantifies the amount of power consumption increased by traffic load x in BS b, and the function $g^b(x)$ is, in BS b, the sum of the traffic load x and the increased energy consumption due to x. Note that $\Delta\mathcal{E}^b(\rho_n^b)$, when the MNO n's traffic is served at BS b, is due to the following algebra:

$$\begin{aligned}\Delta\mathcal{E}^b(\rho_n^b) &= \mathcal{E}^b(\rho^b) - \mathcal{E}^b(\rho_{-n}^b)\\ &= \beta^b E^b \Big\{ \sum_{m\in\mathcal{M}} \rho_m^b - \sum_{m\in-n} \rho_m^b \Big\} = \beta^b E^b \rho_n^b. \end{aligned} \tag{6}$$

We assume that each MNO charges the other MNOs for the prime cost of roaming, even if the MNO does not take a profit margin (i.e., $k_{m(b)} = 0$) due to rationality of the MNO. In the profit margin, we consider that each MNO charges fee for roaming proportional to the increment in service time (BS load) and BS power consumption. Note that since the BS load is the utilization (busy-time) of a BS by queueing theory, the increment in the BS load by roaming can be interpreted as the service time of the roaming traffic.

(iv) Parameter η ***and unit roaming price*** k_m***:*** The parameter $\eta \ge 0$ trade offs QoS and power consumption cost, where larger η implies that MNOs put more emphasis on the power cost in running their network. The bounded value $k_m \in [0, K]$ for some K

[1]We henceforth omit the dependence of the load-related notations on the association vector $\boldsymbol{y}$ for notational simplicity, unless explicitly needed.

[2]In this term, we take into account of all BSs, since under the BS sharing in this paper each MNO potentially uses other MNOs' BSs and all BSs fairly serve all traffics irrespective of their original subscription.

is referred to as *unit roaming price* that can be chosen by the MNO m as a strategy.

Our approach: Time-scale decomposition. In this paper, we do not directly analyze the game in (2), but decompose it into two games, motivated by what happens in practice. Typically, traffic arrival and departure process for a user varies much faster than the overall traffic patterns in a BS, as reported by measurement-based studies [5, 21], which can be considered as a constant during a certain period, e.g., an hour. In contrast, the flow-level dynamics due to flow arrivals/departures are usually less than several minutes. Also, it seems natural to assume that the time scales of price decision and user associations are similar to those of traffic patterns and flow-level dynamics, respectively. In fact, user association runs at a fast time-scale in practice, e.g., amount 100 msec in LTE systems [22].

Inspired by this practice, we separately consider the following two games: *pricing decision* game and *user association* game, where the latter is played for a fixed pricing vector and the former is played under the achieved equilibrium of the latter game played faster. This time-scale separation not only reflects the practice well, but also allows analytical tractability.

User association game. Assuming that $(k_m : m \in \mathcal{M})$ is fixed, each MNO m plays the game with its strategy $\boldsymbol{\rho}_m$ with the payoff function,

$$\textbf{UA-G:} \qquad \mathcal{U}_m(\boldsymbol{\rho}) \ \doteq\ \mathcal{U}_m(k_m, k_{-m}, \boldsymbol{\rho}_m, \boldsymbol{\rho}_{-m}). \qquad (7)$$

Pricing decision game. Each MNO m plays the game with its strategy k_m, where each MNO assumes that if $\boldsymbol{k} = (k_m : m \in \mathcal{M})$ is played, its corresponding equilibrium (if it exists) is immediately obtained as $\boldsymbol{\rho}^*_m(\boldsymbol{k}), \boldsymbol{\rho}^*_{-m}(\boldsymbol{k})$. Thus the payoff function becomes:

$$\textbf{PD-G:} \quad \mathcal{U}_m(k_m, k_{-m}) \doteq \mathcal{U}_m(k_m, k_{-m}, \boldsymbol{\rho}^*_m(\boldsymbol{k}), \boldsymbol{\rho}^*_{-m}(\boldsymbol{k})). \quad (8)$$

4. USER ASSOCIATION GAME

In this section, we first analyze the user association game with a fixed pricing vector. Our primary interests include the existence and the uniqueness of the Nash equilibrium as well as the existence of distributed user association algorithm that converges to the NE. A distributed algorithm for the user association is important in practice, since a centralized user association (where BSs are seriously involved) may required a significantly large amount of signaling message exchanges.

4.1 Equilibrium Analysis

We first prove that the user association game is a potential game, as stated in Theorem 4.1.

THEOREM 4.1. *The user association game is an exact potential game with the following potential function $V(\boldsymbol{\rho})$:*

$$V(\boldsymbol{\rho}) = \sum_{b \in \mathcal{B}} \Big\{ \phi_\alpha(\rho^b) + \eta \mathcal{E}^b(\rho^b) + k_{m(b)} \sum_{\substack{n \in \mathcal{M}, \\ n \neq m(b)}} g^b(\rho^b_n) \Big\}. \quad (9)$$

PROOF. By the definition of a potential game, it suffices to show that the gradient of payoff function is equal to that of the potential function. The gradient of payoff function for all $m \in \mathcal{M}$ is given by:

$$\nabla_{\boldsymbol{\rho}_m} \mathcal{U}_m(\boldsymbol{\rho}) = \Big(\frac{\partial \mathcal{U}_m(\boldsymbol{y})}{\partial \rho^b_m} : b \in \mathcal{B} \Big). \qquad (10)$$

Under the fixed unit roaming price $\boldsymbol{k}$, for all $m \in \mathcal{M}$, we get:

$$\frac{\partial V(\boldsymbol{\rho})}{\partial \rho^b_m} = \frac{\partial \mathcal{U}_m(\boldsymbol{\rho})}{\partial \rho^b_m} = \begin{cases} \frac{1}{(1-\rho^b)^\alpha} + \eta \beta^b E^b, & \text{if } b \in \mathcal{B}_m \\ \frac{1}{(1-\rho^b)^\alpha} + \eta \beta^b E^b (1 + k_{m(b)}) \\ +k_{m(b)}, & \text{if } b \in \mathcal{B}_{-m} \end{cases}$$

Therefore, $\nabla_{\boldsymbol{\rho}_m} \mathcal{U}_m(\boldsymbol{\rho}) = \nabla_{\boldsymbol{\rho}_m} V(\boldsymbol{\rho})$, which completes the proof. □

It is well-known by [23], using the property of a potential game, an NE $\boldsymbol{\rho}^*$ of the user association game should be the solution of the following optimization problem:

$$\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho} \in \mathcal{F}} V(\boldsymbol{\rho}), \qquad (11)$$

The fact that the user association game is a (exact) potential game helps a lot with studying the existence and the uniqueness of NE, as stated in Theorem 4.2.

THEOREM 4.2. **UA-G** *has a pure NE, which is unique for all $\alpha > 0$.* [3]

PROOF. We will prove the following two: (i) the feasible load set $\mathcal{F}$ is a non-empty, compact, and convex set, and (ii) the optimization problem in (11), is a convex program, and the objective function is strictly convex function for all $\alpha > 0$.

First, (i) holds by Lemma 1 in [3]. Second, for (ii), we note that the $\phi_\alpha(\rho^b)$, and $\eta \mathcal{E}^b(\rho^b)$ are strictly convex function and linear function, $\forall \rho^b \in \mathcal{F}$ and $\forall \alpha > 0$, respectively. Further, the roaming cost function $g^b(\rho^b_m)$ is a linear function on $\rho^b_m, \forall m \in \mathcal{M}$. Therefore, the problem (11) is a convex optimization problem with the strictly convex objective function by convex preserving operations. From (i) and (ii) there exist a unique pure NE by [24, 25]. This completes the proof. □

4.2 Distributed User Association

In this subsection, we aim at developing a distributed user association algorithm that provably converges to the NE analyzed in the previous section.

Algorithm. We first describe our association algorithm, which is split into the parts by users and base stations, followed by its rationale in the context of the dynamics in game theory. Our algorithm is inspired by an approximate version of Jacobi play [26], as discussed later.

DUA (Distributed User Association) Algorithm

User algorithm. At every iteration step t, each user receives the followings through broadcast messages from its serving BS i[4]: (i) the vector of unit roaming price $\boldsymbol{k}$, (ii) the load of its serving BS i denoted by $\rho^{i,t}$, and (iii) the load vector of the neighboring BSs of BS i denoted by $\boldsymbol{\rho}^{\mathcal{N}(i),t}$, where $N(i)$ is the set of the neighboring BSs of BS i.

Then, each user associates with a BS that satisfies the following:

$$\arg\min_{b \in \{i\} \cup \mathcal{N}(i)} \frac{1}{c^b_x} \Big\{ \frac{1}{(1-\rho^{b,t})^\alpha} + \eta \beta^b E^b (1 + k(b,m)) + k(b,m) \Big\}, \qquad (12)$$

[3]For $\alpha = 0$, the potential function becomes linear, in which case only existence of NE is guaranteed.

[4]In LTE standards, there is a broadcast control channel (BCCH) in downlink channel structure.

where $k(b,m)$ represents the roaming price of the provider of b when m is not the b's owner and 0 otherwise, i.e.,

$$k(b,m) = \begin{cases} k_{m(b)}, & \text{if } m \neq m(b) \\ 0, & \text{otherwise.} \end{cases}$$

BS algorithm. After each user selects its association at iteration t, each BS receives $\boldsymbol{k}$ and $\boldsymbol{\rho}^{\mathcal{N}(b),t+1}$ from its neighboring BSs, and updates $\rho^{b,t+1}$ based on the users' association decision. Then, each BS advertises the unit roaming price $\boldsymbol{k}$, $\boldsymbol{\rho}^{\mathcal{N}(b),t+1}$, and $\rho^{b,t}$ to all (associated) users, and send $\rho^{b,t}$ to its neighboring BSs.

$\rho^{b,t+1}$ is updated as follows.

$$\rho^{b,t+1} \quad = \quad \omega^t \rho^{b,t} + (1-\omega^t) T(\boldsymbol{y}^{b,t}), \tag{13}$$

where the $\omega^t \in [0,1)$ is an exponential moving average parameter, and $T(\boldsymbol{y}^{b,t})$ is defined as:

$$T(\boldsymbol{y}^{b,t}) \doteq \min \Big\{ \int_{\mathcal{L}} \varrho_m^b(x) \cdot y_m^{b,t}(x) dx, 1-\epsilon \Big\}, \tag{14}$$

where the association $y_m^{b,t}(x)$ is determined by the user at location x as in (12). Note that each BS can easily compute $T(\boldsymbol{y}^{b,t})$ by simply measuring its utilization.

Convergence. We shall prove the convergence of the **DUA**. In the view of potential game, finding an NE of our game is equivalent to solving an optimization problem in (11). Thus, we take the optimization problem to prove the convergence of the **DUA**.

THEOREM 4.3. **DUA** *converges to the unique NE of the user association game in* (7).

PROOF. In the proof, we will exploit the following two: (i) the *user algorithm* (12) gives a descent direction on $V(\boldsymbol{\rho}^t)$ for $\boldsymbol{\rho}^t \in \mathcal{F}$, and $\boldsymbol{\rho}^t \neq \boldsymbol{\rho}^*$, where $\boldsymbol{\rho}^t = (\rho^{b,t} : b \in \mathcal{B})$, and (ii) for $\boldsymbol{\rho}^t \in \mathcal{F}$, there exist $\omega^t \in [0,1)$ such that $V(\boldsymbol{\rho}^{t+1}) < V(\boldsymbol{\rho}^t)$. Here, we do not describe the details of the proof (i) and (ii) due to shortage of space. But the proof is done in the similar way to Lemma 3 and Lemma 4 in [3], respectively. By the (ii), we can select the ω^t that makes the sequence of the **DUA** (i.e., $\boldsymbol{\rho}^t$) monotonically decreasing in t, and then, the sequence must converge to a fixed point by lower bound of $\mathcal{F}$.

Since there is no descent direction for all $\boldsymbol{\rho} \in \mathcal{F}$ at the convergence point by (i), the point is an optimal of the potential function and is the unique NE of our game by (11). □

We remark that our algorithms extend the ones for non-BS sharing cases, e.g., [3, 17, 18]. In particular, our algorithm is similar to those in [17, 18], with a slight difference that we additionally consider unit roaming price and power consumption for the BSs of other MNOs.

Rationale. It is well known that a better reply path converges to the NE in potential games [23]. Thus, to find a dynamic algorithm that converges the NE in **UA-G**, we first consider *Jacobi play* [26] which is one of the algorithms that provide a better reply path by taking exponential-weighted moving average of the *best response.* However, Jacobi play is impractical in our case due to the hardness of finding the best response, as discussed later. Thus, we take an approximated Jacobi play that provably converges to the NE with implementable hardness, and propose the **DUA** that practically implements the approximated Jacobi play in cellular networks.

In order to explain the hardness of Jacobi play, we first describe the Jacobi play as follows.

$$J_m(\boldsymbol{\rho}^t) \doteq \omega^t \boldsymbol{\rho}_m^t + (1-\omega^t) B_m(\boldsymbol{\rho}^t), \tag{15}$$

where the $B_m(\boldsymbol{\rho}^t)$ is the best response described as follows.

$$B_m(\boldsymbol{\rho}_{-m}^t) \doteq \underset{(\boldsymbol{\rho}_m, \boldsymbol{\rho}_{-m}^t) \in \mathcal{F}}{\arg\min} \; \mathcal{U}_m(\boldsymbol{\rho}_m, \boldsymbol{\rho}_{-m}^t), \tag{16}$$

where $\boldsymbol{\rho}_{-m}^t$ is the strategies of the others at iteration step t.

To calculate (15), the best response $B_m(\cdot)$ should be available, but finding it turns out to be very complexity due to its complex necessary condition, given by:

$$\langle \nabla_{B_m(\boldsymbol{\rho}_{-m}^t)} \mathcal{U}_m(B_m(\boldsymbol{\rho}_{-m}^t), \boldsymbol{\rho}_{-m}^t), (\boldsymbol{\rho}_m - B_m(\boldsymbol{\rho}_{-m}^t)) \rangle \geq 0 \tag{17}$$

The inner product in the necessary condition (17) can be computed as follows.

$$\begin{aligned} &\langle \nabla_{B_m(\boldsymbol{\rho}_{-m}^t)} \mathcal{U}_m(B_m(\boldsymbol{\rho}_{-m}^t), \boldsymbol{\rho}_{-m}^t), (\boldsymbol{\rho}_m - B_m(\boldsymbol{\rho}_{-m}^t)) \rangle \\ &\qquad = \sum_{b \in \mathcal{B}} \Big\{ \psi^b(\rho^{b,B})(\rho_m^b - B_m^b(\boldsymbol{\rho}_{-m}^t)) \Big\} \\ &\qquad = \sum_{b \in \mathcal{B}} \Big\{ \psi^b(\rho^{b,B}) \int_{\mathcal{L}} \varrho_m^b(x)(y_m^b(x) - y_m^{b,B}(x)) dx \Big\} \\ &\qquad = \int_{\mathcal{L}} \sum_{b \in \mathcal{B}} \Big\{ \varrho_m^b(x) \psi^b(\rho^{b,B})(y_m^b(x) - y_m^{b,B}(x)) \Big\} dx, \end{aligned} \tag{18}$$

where $\rho^{b,B} = B_m^b(\boldsymbol{\rho}_{-m}^t) + \rho_{-m}^{b,t}$, and the $B_m^b(\boldsymbol{\rho}_{-m}^t)$ is the element of $B_m(\boldsymbol{\rho}_{-m}^t)$ for BS b, i.e, $B_m(\boldsymbol{\rho}_{-m}^t) = \rho_m^{b,t+1}$. For all $b \in \mathcal{B}$, $\psi^b(\rho^{b,B})$ is defined as:

$$\psi^b(\rho^{b,B}) \doteq \frac{1}{(1-\rho^{b,B})^\alpha} + \eta \beta^b E^b (1 + k(b,m)) + k(b,m). \tag{19}$$

Then, the necessary condition (17) holds when the following condition is satisfied.

$$\sum_{b \in \mathcal{B}} \varrho_m^b(x) \psi^b(\rho^{b,B})(y_m^b(x) - y_m^{b,B}(x)) \geq 0, \quad \forall x \in \mathcal{L}. \tag{20}$$

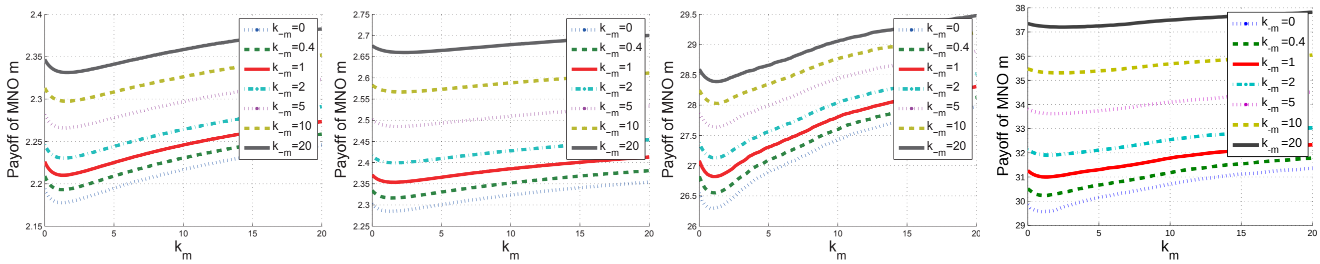
In order to satisfy (20), in the best response, each MNO plays a deterministic user association such that each user selects a BS with probability 1 which satisfies the following condition:

$$\underset{b \in \mathcal{B}}{\arg\min} \frac{1}{c_x^b} \Big\{ \frac{1}{(1-\rho^{b,B})^\alpha} + \eta \beta^b E^b (1 + k(b,m)) + k(b,m) \Big\}. \tag{21}$$

The difficulty in the best response is caused by $\frac{1}{1-\rho^{b,B}}$ in (21). Finding the user association that satisfies (21) is a fixed point problem due to the definition of $\rho^{b,B}$, requiring an exhaustive search. However, since the user association can vary within tens of milliseconds, the *best response* is not implementable. So, we consider an *approximate version of best response* such as the *user algorithm* (12) motivated by Jacobi play. The key idea in the user algorithm is that if $\rho^{b,B}$ is given, then the best response (21) is easily solvable. So, we approximate $\rho^{b,B}$ to the value at the previous iteration step t (i.e., $\rho^{b,t}$), in the user algorithm (12).

5. PRICING DECISION GAME

In the previous section, we assume a fixed pricing vector, and then study the game of how to distribute the loads (via user association) of a MNO to other MNOs. In this section, we study how strategically MNOs decide on their pricing vectors, i.e., $\boldsymbol{k}$. Recall that when a strategy vector $\boldsymbol{k}$ is played, our time scale separation assumption gives us the loads at NE, $\boldsymbol{\rho}^*(\boldsymbol{k})$, (from the corresponding association game), which in turn is included in the payoff function



(a) Case A in the toy example

(b) Case B in the toy example

(c) Real deployments with symmetric traffic conditions

(d) Real deployments with asymmetric traffic conditions

Figure 2: 2 MNOs. we plot the payoff function of MNO m for various values of MNO $-m$'s prices.

of the pricing decision game (**PD-G**), as seen in (8). This nontrivial mapping from $\boldsymbol{k}$ to $\boldsymbol{\rho}^*(\boldsymbol{k})$ makes the analytical study highly challenging, for which we rely on numerical analysis.

5.1 Equilibrium Analysis

Quasi-convexity of payoff functions. We first numerically show that the payoff function is a quasi-convex function, and under such a conjecture, we prove the existence of pure NE as stated in Theorem 5.1. Figure 2 (The setup details will be explained in Section 6) plots the shape of the payoff function in (8) for varying k_m for a variety of k_{-m} values, where we assume two MNOs. As we see, although in numerical simulations, the payoff function seems quasi-convex. Formally proving its quasi-convexity for a general number of MNOs is left as a future work. Instead, we provide an intuitional explanation for quasi-convexity next.

The intuition behind quasi-convexity is similar to the law of supply and demand in microeconomics. Suppose that $k_m = 0$, then since the MNO m does not charge the roaming fee to roamed traffic, the roamed traffic will grow up, in which case MNO m's QoS cost would also increase, resulting in the increase of the m's payoff (recall that our payoff function is a cost function). In contrast, when k_m is large, there will be almost no roamed traffic, where no gain from roaming is obtained, and thus a large payoff. Note that the economic roaming gain comes from $x \times y$, where x is the roaming fee and y is the amount of roamed traffic. However, whenever (i) we slightly increase the roaming fee from a very small one, or (ii) we slightly decrease it from a very large one, the larger benefit can be achieved, either x increases in (i) or y increases in (ii).

The following theorem states the existence of a pure NE.

THEOREM 5.1. *Under the assumption of quasi-convexity of the payoff function in* (8), **PD-G** *has a pure NE.*

PROOF. Each $k_m \in [0, K]$ for all $m \in \mathcal{M}$ is closed and bounded, and is obviously convex set. $\mathcal{U}_m(k_m, k_{-m})$ is continuous in k_{-m}, and quasi-convex in k_m from the hypothesis. Thus, **PD-G** has a pure NE by Theorem 1.2 in [27]. ☐

5.2 Efficiency of Strategic BS Sharing

To investigate the efficiency of BS sharing where MNOs strategically behave, we define a measure that compares it to the case when the MNOs are fully cooperative (i.e., $k_m = 0$, $\forall m \in \mathcal{M}$). In full-cooperation, it seems natural that the entire collection of MNOs would assign the loads so that it maximizes the total aggregate happiness of the MNOs, i.e., $\min_{\boldsymbol{\rho}\in\mathcal{F}} \mathcal{V}_G(\boldsymbol{\rho})$, where

$$\mathcal{V}_G(\boldsymbol{\rho}) \doteq \sum_{b\in\mathcal{B}} \Big\{ \phi_\alpha(\rho^b) + \eta \mathcal{E}^b(\rho^b) \Big\}. \tag{22}$$

This measure is chosen based on the following: From the perspective of MNOs that are fully cooperative, users can fairly use all BSs, irrespective of the ownership of the BSs. Therefore, an ideal operation of cooperative MNOs would be the one that assumes that a virtual MNO owns the entire BSs.

Then, the efficiency of sharing BSs for strategic MNOs is measured by the following ratio:

$$\frac{\min_{\boldsymbol{\rho}\in\mathcal{F}} \mathcal{V}_G(\boldsymbol{\rho})}{\mathcal{V}_G(\boldsymbol{\rho}^*(\boldsymbol{k}^*))}, \tag{23}$$

where the $\boldsymbol{\rho}^*(\boldsymbol{k}^*)$ is the NE of the user association game when the NE $\boldsymbol{k}^*$ of the pricing decision game is applied.

6. NUMERICAL ANALYSIS

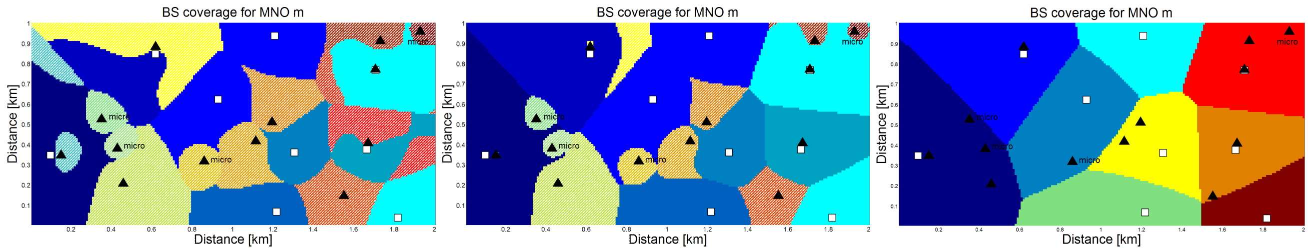
In this section, we show the various aspects of the NE through extensive simulations. In all simulations, we consider a duopoly market, in which 2 MNOs denoted by m and n share their BSs with each other in the real 3G deployment as in Figure 1. In the payoff function, we use $\eta = 10^{-5}$ for all simulations. In the power consumption model, we consider that each macro and micro BS has $E^b = 865$ W and $E^b = 38$ W, respectively, and $\beta^b = 0.5$, as in [28]. We assume that all users are uniformly distributed in the rectangular domain. For all points $x \in \mathcal{L}$, we assume that a file request has exactly one file whose size is log normally distributed with mean $1/\mu(x) = 100$ kbytes, and the mean arrival rate of file transfer is $\lambda_m(x) = 1.859 \times 10^{-4}$, excluding the asymmetric traffic conditions. We apply an urban cell path loss model $35.2 + 35log(d) + 26log(f/2)$ in IEEE 802.16m in [29], where d is the distance between the user and the BS, and $f_m = 2.154$ GHz, and $f_n = 2.141$ GHz[5] are the center carrier frequencies of MNO m and n, respectively. We assume that each MNO has 10 MHz bandwidth for the downlink. We apply interference only on the BSs of the same MNO.

BS coverage. We first show that our user association algorithm (**DUA**) appropriately determines the coverage of BSs with minimizing QoS cost, BS power consumption, and roaming fee under various pricing schemes. In simulations, we consider that 3-pair of unit roaming price (k_m, k_n) such as $(0, 0)$, $(1.06, 1.28)$[6], and $(200, 200)$, which represent full-cooperation, NE, and non-BS sharing scenario, respectively.

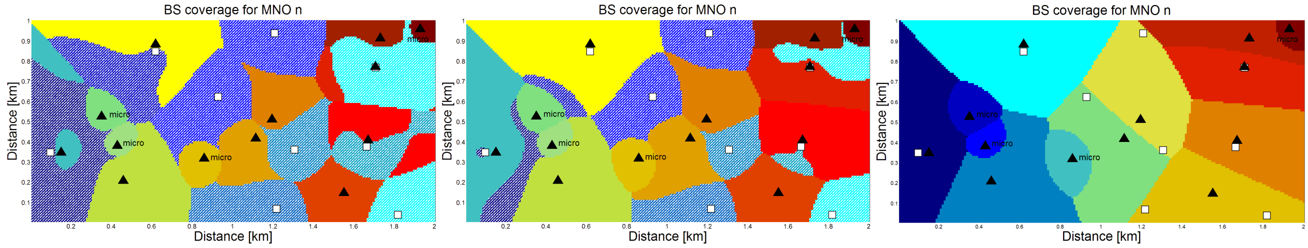
Figure 3 shows that each MNO tends to become more conservative on roaming as the unit roaming price of the other MNO increases. Note that in Figure 3, the BSs of MNO m, and n are denoted by white rectangle and black triangle, respectively, and if the BS is a micro BS, then we mark 'micro' nearby the BS. The others are macro BSs. The shaded region represents the coverage by the

[5]The frequency 2.154 GHz and 2.141 GHz are the center frequencies of two major MNOs in UK, respectively.

[6]Here, the pair (1.06,1.28) is a NE of the pricing decision game from our numerical computation.



(a) Coverage for m in full-cooperation (b) Coverage for m in NE (c) Coverage for m in non-BS sharing



(d) Coverage for n in full-cooperation (e) Coverage for n in NE (f) Coverage for n in non-BS sharing

Figure 3: BS coverage for various roaming prices

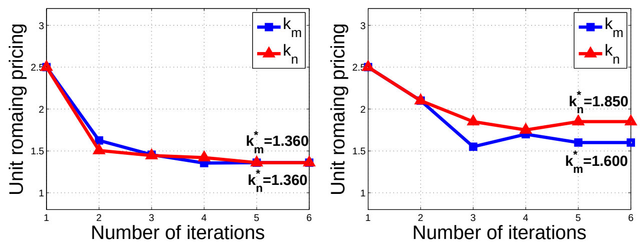
BSs of other MNO (i.e., the roaming region) in Figure 3. Specifically, in the full-cooperation, the BSs of MNO m and n fairly serve all traffic as shown in Figures 3(a), 3(d). As $\boldsymbol{k}$ increases, each MNO avoids roaming its traffic due to expensive roaming fee, and finally the user association game reaches to non-BS sharing (see Figures 3(c), 3(f)) for sufficiently large $\boldsymbol{k}$. Interestingly, BS sharing gives a chance for users to be associated with a BS which gives low-interference. Since high-interference gives low-data rate for a user and it turns out to increase the QoS cost (3) (delay) of serving BS, the user may tend to be associated with a BS that gives low-interference among the BSs of different MNOs by **DUA**. As shown in Figures 3(a), 3(b), 3(d), 3(e), each MNO aggressively uses the BSs of the other MNO in the region where the BSs of the MNO are closely located to each other (i.e., in heavy interference region).

Unit roaming price. We next study the impact of the number of BSs and the number of subscribers in each MNO on the NE. To that end, we consider three toy examples with different traffic and infrastructure conditions as described in what follows:

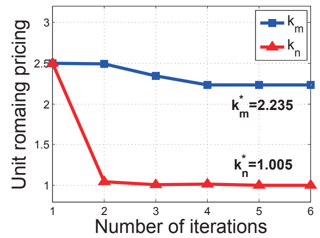
- **Case A:** Both MNOs have exactly one BS each, and the traffic conditions are same. The BSs are located at $(0, 0)$ and $(2, 2)$ in 2 km by 2 km rectangular area, respectively. The subscribers uniformly generate the traffic with $\lambda_m(x) = \lambda_n(x) = 1.46 \times 10^{-7}$.
- **Case B:** The BS environment is as same as in case A, but the traffic conditions are different, i.e., $\lambda_m(x) = 2\lambda_n(x) = 2.92 \times 10^{-7}$. Here, the difference in the generated traffic means that the difference in the number of subscribers with an assumption that each subscriber uniformly generates the traffic.
- **Case C:** The traffic conditions of each MNO are the same, but the BS deployment is heterogeneous. MNO m has additional BS in $(0, 2)$, and $\lambda_m(x) = \lambda_n(x) = 1.46 \times 10^{-7}$.

For all simulations, we take the *best response dynamic* to find the NE of the pricing decision game (**PD-G**), thus the convergence point is exactly an NE by definition of NE.

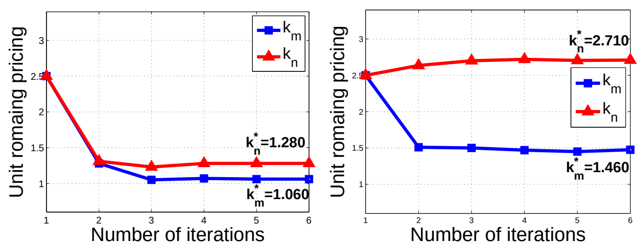
In case A, each MNO has the same unit roaming price at NE as shown in Figure 4(a) due to the symmetric conditions on traffic and the number of BS. However, in case B, the difference in traffic increases the roaming price of MNO n who has less traffic than the others as shown in Figure 4(b). Since the MNO n has fewer subscribers than MNO m, the BS of MNO n has lower load than



(a) Case A in the toy example (b) Case B in the toy example

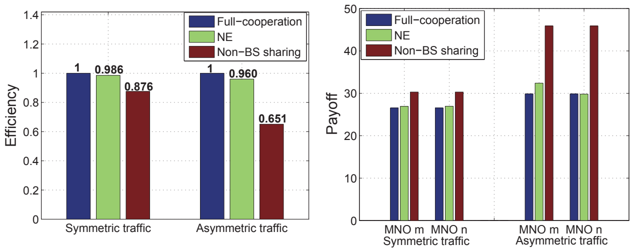


(c) Case C in the toy example



(d) Real deployment with symmetric traffic conditions (e) Real deployment with asymmetric traffic conditions

Figure 4: Equilibrium roaming prices for various cases



(a) Comparison with full-cooperation in (23)

(b) Payoff

Figure 5: Efficiency of strategic BS sharing

the BS of MNO n, thus, the unit roaming price of MNO n as well as the demands for roaming to MNO n grow. In case C, the unit roaming price of MNO m is twice higher than that in case A as shown in Figure 4(c). The added BS of MNO m gives better channel conditions to the users who are near the BS, and then, not only the BS load of MNO m is reduced, but also demands for roaming of MNO n increase, resulting in the increase of the unit roaming price of MNO m. We perform simulations in real 3G deployment and get similar results as toy examples. Like in the toy examples, we conduct simulations in two different environments, symmetric and asymmetric traffic conditions. For the symmetric traffic conditions, the NE is (1.06, 1.28) due to the difference in the number of BSs as shown in Figure 4(d). In Figure 4(e), we consider asymmetric traffic conditions and give more subscribers to MNO m such as $\lambda_m(x) = 2\lambda_n(x)$, and then, k_n^* is increased due to the increasing demands for roaming to MNO n.

Efficiency. Figure 5 shows the performance of our game in terms of efficiency (23) and the payoff of each MNO in real 3G deployment. As shown in Figure 5(a), the NE achieves almost the efficiency of full-cooperation for all cases consistently. However, conventional non-BS sharing gives the lowest efficiency for all cases and the efficiency is getting worse in the asymmetric traffic conditions. Moreover, the NE gives the better payoff than non-BS sharing. As shown in Figure 5(b), the payoff of NE is dramatically reduced and it is about 65-89% of the payoff in non-BS sharing scenario. Especially, in the asymmetric traffic conditions, the MNO n gets the smallest payoff at NE due to the better roaming income than full-cooperation. Thus, the rationality of each MNO is guaranteed in the BS sharing, since the BS sharing always gives the smaller payoff to all MNOs.

7. CONCLUSIONS

In this paper, we studied the impact of sharing BSs by formulating two forms of games, where one is the pricing decision game and the other is user association game, each of which is assumed to played at different time scales. We demonstrate that there exists a significant degree of energy saving, once an appropriate competition rule is provided, where more rigorous analysis of the pricing decision game is left as a future work.

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