Energy-efficient User Association in Cellular Networks: A Population Game Approach

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***Abstract*—In this paper, we address the problem of associating mobile stations with base stations (BSs) in an energy-efficient manner. We take the population game approach, which allows tractable analysis of many selfish mobiles without growing mathematical complexity, where our study provides two practical implications on energy-efficient BS associations: (i) how to control so-called association pricing so that an entire cellular network is operated with the goal of optimizing a social objective, and (ii) how to develop distributed, energy-efficient association algorithms. To that end, we first define a game, where mobile stations are the players, and their association portion for different base stations are their strategies. Then, from our equilibrium analysis, we prove that a simple power-dependent pricing by operators leads Nash equilibrium to be equal to the optimal solution of a social optimization problem (i.e., zero price-of-anarchy). Next, we study three evolution dynamics of mobile stations, each expressed as a differential equation, and connect each of them to a distributed association control mechanism, where three dynamics provably or experimentally converge to the Nash equilibrium (which is equal to the socially optimal point).**

I. INTRODUCTION

In response to high data demand in cellular systems, *user association problem*, the problem of associating a mobile station (MS) [1] with an appropriate base station (BS) is of prime importance. It has been evidenced in literature that a simple approach of connecting an MS to the BS providing the highest received signal strength has a lot of performance problems due to its load-agnostic behavior. In fact, the user population in a cell has significant impact on the actual individual MS throughput, thereby many load-aware association schemes have been proposed to date [1]–[12].

In addition to performance, energy-efficiency in wireless networks is also an important metric. Especially, recent interests in greening such as the CO_2's potential harms (e.g., global warming) to the environment as well as the economic issues recently motivate a surge of energy-efficient research. There are many components to save energy in cellular networks, ranging from cooler and power amplifiers to dynamic switching on/off of BSs. User association is also highly involved in energy consumption in the network, and the need of user association mechanisms, which is both energy and load aware, grows. There exists a complex interplay between energy-efficiency and performance (such as throughput or delay), often showing a tradeoff. This is because high performance requires load balancing of MSs, whereas energy efficiency increases when MSs are associated with nearby BSs, often suffers from load-imbalance.

In this paper, we study an energy-efficient user association problem from a population game-theoretic perspective. Population game [13] groups the entire MSs into a *finite* number of classes of infinitesimal MSs having similar attributes, e.g., the set of connectable BSs, their link conditions, and the spatial traffic distribution. This enables us to have a mathematically tractable framework without growing mathematical complexity and easily obtain the implications into distributed BS association mechanisms. Our model uses a flow-level dynamic where data traffic is initiated at random and its workload is also random, so that after a random amount of sojourn time in the system it leaves. This flow-level dynamic seems to provide more practical intuitions and results than the statically backlogged setting often taken in other researches. In our flow-level dynamic, we model spatially heterogenous traffic distribution and also capture signal degradation incurred by interference from other BSs.

In our game, we model the payoff function by the combination of the selfish performance objective of users and the cost for using BSs' energy, where an user's selfish performance objective is described by the delay performance conditioned that the user's offering load. We first prove that the population game designed by the aforementioned payoff function becomes a potential game. In potential game, Nash equilibrium (NE) is characterized by the Karush-Kuhn-Tucker (KKT) condition of the potential function, offering an easy path to the equilibrium analysis. Then, we prove that the NE coincides with the socially optimal point, implying there is no price-of-anarchy. This remarkable result stems from a smart association pricing scheme, instilled as a cost part of user's payoff function.

Next, we consider three kinds of evolutionary dynamics, the best response dynamic, replicator dynamic, and Brown-von Neumann-Nash (BNN) dynamic, each of which captures how mobiles evolve over the system state changes. As studied in literature, the best response and BNN dynamics provably con-

[1] We use 'user', 'MS', and 'players' interchangeably throughout this paper.

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verge to the NE. Unfortunately, the replicator dynamic may not converge to a stationary point that is not NE, but under some reasonable conditions of initial points, we experimentally show that the replicator dynamic is also highly likely to converge to NE. Three dynamics were originally developed to model selfish players in population games, but we connect them to energy-efficient, distributed association control algorithms. In fact, we show that a distributed algorithm developed from an optimization's perspective can be reverse-engineered by a game-theoretic approach, with more practical advantages being obtained in the algorithm derived from our population game approach.

Related work

Recently, the authors in [10] formulate an optimization problem that trade offs performance and energy efficiency, and study both energy-efficient association and dynamics BS on/off switching. They use a time-scale separation between association and on/off operations, enabling two different problems. The social objective function in our paper is equivalent to that in [10] without BS dynamic on/off switching. However, our paper significantly differs from [10] in that we approach the problem from the game-theoretic perspective. Specifically, using a population game framework, we consider a *finite* number of classes in describing different heterogenous traffic characteristics and a discrete set of MS data rates; in [9], [10], it was assumed that there exist number of classes and data rates are continuous for simplicity. However, this simplification does not capture the real systems well, and the deterministic user association [9], [10] does not generally achieve optimality with a finite number of classes in practice. This is because the cell boundary should be a *region*, not a *line* as in [9], [10] when adaptive modulation and coding (AMC) is employed. The area of cell boundary can be very large, and thus probabilistic user association is required to achieve optimality without causing ping-pong effect. This becomes particularly important when the traffic load is high, where the regime when the load balancing is indeed of paramount importance. We show that our algorithm motivated by the best response dynamic resolves this issue and substantially improves the performance. See Section IV for more details.

There exists work, see e.g., [11], [14], [15] that studies a BS/WLAN association problem in a game-theoretic setting. [14] used the user performance of UDP/TCP throughput with varying frame length over WLAN access points, whereas we use flow-level delay as a performance metric which depends on BS load. The authors in [11] suggested the general concave utility function, formulated a game, and proved that the total utility is maximized at the Nash equilibrium. The work in [15] studied a Stackelberg game between BS placement and user association. The main difference from the above lies in that we consider both energy-efficiency and flow-level dynamic using a population game.

Other related work includes [16], which studies the load balancing problem among server farms (where a server can be considered as a BS in our case) using game theory. In [16], the authors assumed a fixed processing capacity of each server and the capacity does not depend on users. We model spatially heterogeneous users, and thus BS-user capacity should differ across users, which makes the problem much more challenging. There exists an array of research on BS load-balancing. Some of earlier studies assumed a centralized processor that establishes cell load-balancing [1]–[7]. Due to its weakness in terms of scalability and flexibility, the distributed algorithms were proposed [8]–[10]. We refer the readers to [17] for a list of networking problems analyzed by population game theory. Greening with focusing on dynamic BS on/off switching has been studied in [10], [18]–[21].

II. Preliminaries and Model

A. Population Game

Basic concepts. We briefly provide the basics of population game, which we refer the readers to [13] for more details. A *population game* F is defined by the *society* of continuous mass of user groups called *classes*. Denote the set of classes by $\mathcal{Q}$ and the number of classes by Q. Each class $q \in \mathcal{Q}$ has continuous *mass* d^q. Each class q has its own strategy sets $\mathcal{S}^q = \{1, ..., S^q\}$. A single entity in the class is called *player*, and each player in class q selects its own strategy among the strategy set $\mathcal{S}^q$. The *state* of class q is defined as its distribution of strategic decisions, denoted as $y^q = [y_1^q, ..., y_{S^q}^q]$, where y_i^q represents the mass of players in class q who plays strategy $i \in \mathcal{S}^q$. The set of states of class q is denoted as $\mathcal{Y}^q = \{y^q \in \mathbb{R}_+^{S^q} \,|\, \sum_{i\in\mathcal{S}^q} y_i^q = d^q\}$. The social state $\boldsymbol{y} = [y^1, ..., y^Q]$ is simply a cartesian product of the class states. Again, the set of all possible social states is denoted as $\mathcal{Y} = \prod_{q\in\mathcal{Q}} \mathcal{Y}^q$. The marginal payoff function F_i^q per unit mass of each class q for each strategy i is defined on each social state. Thus, we have a collection of marginal payoff functions $F = (F_i^q : i \in \mathcal{S}^q, q \in \mathcal{Q})$. We also use F to name a population game for notional simplicity. Each player in each class receives its payoff depending on its own strategic decision. The aggregate payoff of class q is $\sum_{i\in\mathcal{S}^q} y_i^q F_i^q(\boldsymbol{y})$.

Best response and Nash equilibria. A solution concept that is generally used in game theory is called *Nash equilibrium*. This notion is also used in population games. We first define the concept of *best response correspondence*, which means a set of selfishly optimal strategies given a social state. In class q, the pure best response correspondence $b^q : \mathcal{Y} \to \mathcal{S}^q$ is defined by $b^q(\boldsymbol{y}) = \arg\max_{i\in\mathcal{S}^q} F_i^q(\boldsymbol{y})$. Also, the mixed best response correspondence for class q is defined as $B^q(\boldsymbol{y}) = \{x^q \in \mathcal{Y}^q : x_i^q > 0 \to i \in b^q(\boldsymbol{y})\}$.

Definition 1: A social state $\boldsymbol{y} \in \mathcal{Y}$ is a *Nash equilibrium* of the population game F if every player in the society is choosing the best response of $\boldsymbol{y}$, i.e., the set of all Nash equilibria $\mathrm{NE}(F)$ is:

$$\mathrm{NE}(F) \;=\; \{\boldsymbol{y} \in \mathcal{Y} : y^q \in B^q(\boldsymbol{y}) \text{ for all } q \in \mathcal{Q}\}.$$

Potential game. If the payoff function F has a special form, the characterization and analysis of Nash equilibria becomes much more tractable.

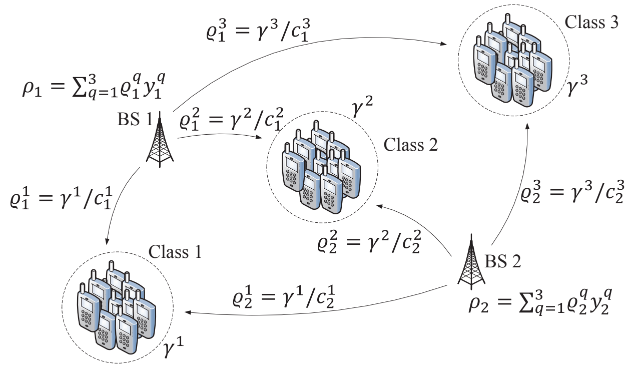


Fig. 1: System Model and Notations.

Definition 2: A population game F is a *potential game* if there exists a C^1 function called *potential function* $\Phi : \mathcal{Y} \to \mathbb{R}$, satisfying $\frac{\partial \Phi}{\partial y_i^q}(\mathbf{y}) = F_i^q(\mathbf{y})$, i.e., $\nabla\Phi(\boldsymbol{y}) = F(\boldsymbol{y})$, for all $\mathbf{y} \in \mathcal{Y}, i \in \mathcal{S}^q$, and $q \in \mathcal{Q}$.

In the potential game, the strategic improvement of users increases potential function. Thus, at the local maxima of the potential function, there exist no incentives for each player to deviate from its own decision. In other words, the local maximum of the potential function is equivalent to a Nash equilibrium, as summarized as the following theorem [13]:

Theorem 1: If a population game F is a potential game with the potential function Φ, then $\text{NE}(F) = \text{KKT}(\Phi)$, where $\text{KKT}(\Phi)$ is the set of points satisfying the KKT condition of Φ.

B. System Model

Network and Users. We consider a cellular network consisting of a set $\mathcal{S}$ of BSs. The society corresponds to the set $\mathcal{Q}$ of all users, composed of a finite set of classes, where a class q is a population of users who commonly share (i) the set $\mathcal{S}_q$ of BSs allowing association to the entities of class q and (ii) the link capacity from each of such BSs, (iii) the traffic characteristic. As mentioned earlier, we assume that each class q has a continuous mass of users, and the class-q mass is denoted by d^q.

Traffic, Capacity, and Load. All users in class q have a Poisson arrival of file transfer requests with rate λ^q, and each file size is independently distributed with mean $1/\mu^q$. Thus, the total request rate of class q is $\lambda^q d^q$. Let the traffic load density of class q per unit mass be $\gamma^q = \lambda^q/\mu^q$. As mentioned earlier, we assume that all users in the same class receive the same link capacity from each BSs. Denote c_i^q the link capacity that each user in class q can achieve from the BS i. Note that c_i^q may differ among all pairs of user classes and BSs and c_i^q can capture the inter-cell interference as well. The *system-load density* ϱ_i^q is defined as the time fraction required by BS i to serve the request from the unit mass of the class q, i.e., $\varrho_i^q = \gamma^q/c_i^q$, where it is assumed $\min_i \varrho_i^q < \infty$, which means there exists at least one BS which provides positive link capacity to the class q and thus can serve the request from the class q. Let ρ_i be the *load* of the BS i. The load ρ_i is represented as the sum of the traffic loads in BS i from all classes, i.e., $\rho_i = \sum_{q\in\mathcal{Q}} \varrho_i^q y_i^q$. The ρ_i can be interpreted as the fraction of time needed in BS i to serve the entire incoming traffic. The load ρ_i should be less than 1 in order to make the system stable, which is assumed in this paper. Fig. 1 visually explains our system model.

BS Energy Model. We model BSs' energy consumption by a combination of the static power consumed whenever a BS is turned on and the load-dependent power, where each portion is tunable by a parameter, as stated next:

$$\text{Energy consumption of BS } i = (1 - q_i)\rho_i P_i + q_i P_i, \qquad (1)$$

where P_i is the amount of energy of BS i when fully utilized and $0 \le q_i \le 1$ is the parameter quantifying the portion of the static power at BS i. For example, the case $q_i = 0$ corresponds to the BS that is entirely *energy-proportional*. In practice, a typical UMTS BS consumes 800-1500W for static power and 20-40W for RF output power [10], and q_i is not close to 0; the range of q_i is roughly 0.5–0.8 in 3G cellular networks [22].

III. Association Game and Equilibrium Analysis

We now define a population game, called *association game*, by completing the model of (marginal) payoff function for each class. Prior to the game description, we first present a social optimization problem that is intended to be solved by a regulator, e.g., an MNO (Mobile Network Operator). We later compare the equilibrium of the defined game and the optimal solution of the social optimization problem.

A. Social Objective

Consider the following optimization problem:

$$\begin{aligned} \text{maximize} \quad & \Phi(\boldsymbol{y}) = \Phi_{F,\alpha}(\boldsymbol{y}) + \eta\Phi_G(\boldsymbol{y}) \qquad (2)\\ \text{subject to} \quad & \rho_i = \sum_{q\in\mathcal{Q}} y_i^q \varrho_i^q < 1 \text{ for all } i \in \mathcal{S} \\ & \text{and } \sum_{i\in\mathcal{S}^q} y_i^q = d^q \text{ for all } q \in \mathcal{Q}, \end{aligned}$$

where the term $\Phi_{F,\alpha}(\boldsymbol{y})$ corresponds to *flow-level performance*, the term $\Phi_G(\boldsymbol{y})$ represents the amount of *energy consumption*, and $\eta \ge 0$ is the parameter that trades off those two metrics.

For the performance term $\Phi_{F,\alpha}(\boldsymbol{y})$ in (2), we take the approach in [9] that parameterizes the flow-level efficiency with α:

$$\Phi_{F,\alpha}(\boldsymbol{y}) = \begin{cases} -\sum_{i\in\mathcal{S}} \frac{(1-\rho_i(\boldsymbol{y}))^{1-\alpha} - 1}{\alpha - 1}, & \alpha \ne 1 \\ -\sum_{i\in\mathcal{S}} \log\left(\frac{1}{1-\rho_i(\boldsymbol{y})}\right), & \alpha = 1. \end{cases} \qquad (3)$$

The parameter α is called *degree of load balancing*. For $\alpha = 0$, the function becomes $\sum_{i\in\mathcal{S}} \rho_i$, hence the users have *rate-optimal* behavior. For $\alpha = 2$, corresponding to *delay-optimal*, the function becomes $\sum_{i\in\mathcal{S}} \frac{\rho_i}{1-\rho_i}$, which is proportional to the average delay of M/GI/1 multi-class processor sharing queue [23]. The second term in (2) represents the total energy consumption of BSs (simply corresponding to a cost term), given by the summation of consumed energy over all BSs:

$$\Phi_G(\boldsymbol{y}) = -\sum_{i\in\mathcal{S}} \left[(1-q_i)\rho_i(\boldsymbol{y})P_i + q_iP_i\right]. \tag{4}$$

Note that we put negative signs to both terms in (2) simply to make the target optimization a maximization problem.

B. User Association Game Formulation

We now design our association game for which we need to define the marginal payoff function for each class q and each strategy available to the class q. Note that the marginal payoff function is interpreted as the payoff obtained by the newcomers in the corresponding class when all other users' strategies are given. We consider the following form of the marginal payoff function:

$$\begin{aligned} F_i^q(\boldsymbol{y}) &= -\left[\frac{\varrho_i^q}{(1-\rho_i(\boldsymbol{y}))^\alpha} + \eta P_i \varrho_i^q (1-q_i)\right] \\ &= -\varrho_i^q\left[(1-\rho_i(\boldsymbol{y}))^{-\alpha} + \eta P_i(1-q_i)\right]. \end{aligned} \tag{5}$$

The payoff function $F_i^q(\boldsymbol{y})$ is structured by two major terms: (i) selfish flow-level utility and (ii) power pricing.

(i) *Selfish flow-level utility.* The first term of (5) denotes the selfish utility motivated by the selfish flow-level performance. For $\alpha = 0$, the first term becomes $\varrho_i^q(=\gamma^q/c_i^q)$, directing users to selfishly prefer the BSs providing high rate without considering the offered load in the associating BS. For $\alpha = 1$, this term becomes proportional to the conditional delay experienced by the users in the class q, where the conditional delay means the delay experienced by an user conditioned on associating with a particular BS, i in this case (see [9], [16] that use the notion of conditional delay under different models for different purposes). As α grows, users increasingly take into consideration the BS loads in association, as the payoff function decades more sharply with increasing ρ_i.

(ii) *Power Pricing.* The second term corresponds to the consumed energy of BS i to serve the users in class q. Note that this term does not depend on the social state, implying that the cost of associating with a particular BS is independent of other class' offered load. Recall that $1-q_i$ is the portion of load-dependent, consumed energy. Thus, $P_i\varrho_i^q(1-q_i)$ corresponds to the consumed energy only by class q, that can be interpreted as the *price* that an user in class q should pay to use BS i's power resource. An interesting feature is that when $q_i = 1$ (energy unproportional), there is no incurred power cost in this marginal payoff function.

C. Equilibrium Analysis and Price-of-Anarchy

In this subsection, we provide the equilibrium analysis of our game. Three main features of our interests are: existence, uniqueness, and Price-of-Anarchy (PoA) of the equilibrium (i.e., NE). Let $y^\star$ and $y^{\rm NE}$ be the socially optimal solution of (2) and an NE (if it exists). Then, in this paper, we define PoA to be $|\Phi(y^\star) - \Phi(y^{\rm NE})|$. In many cases, it is quite challenging and mathematically complex to analyze those three features, especially when the game has a large degree of couplings. However, our game is provably a potential game, opening an easy path to the analysis, as we will henceforth discuss in this subsection.

We first prove that our association game is a potential game. Lemma 1 states that the social objective function in (2) is the potential function.

Lemma 1: The objective function $\Phi(\boldsymbol{y})$ in (2) is a potential function of the population game with the marginal payoff function (5).

Proof: From Definition 2, it suffices to check $\frac{\partial \Phi}{\partial y_i^q}(\boldsymbol{y}) = F_i^q(\boldsymbol{y})$. For the case of $\alpha = 1$,

$$\begin{aligned} &\frac{\partial \Phi}{\partial y_i^q}(\boldsymbol{y}) \\ &= -\frac{\partial}{\partial y_i^q}\left[\sum_{i\in\mathcal{S}} \log\left(\frac{1}{1-\rho_i}\right) + \eta\Phi_G(\boldsymbol{y})\right] \\ &= -\left[(1-\rho_i)\cdot\frac{1}{(1-\rho_i)^2}\cdot\frac{\partial \rho_i}{\partial y_i^q} + \eta(1-q_i)P_i\frac{\partial \rho_i}{\partial y_i^q}\right] \\ &= -\varrho_i^q\left[(1-\rho_i)^{-1} + \eta P_i(1-q_i)\right] = F_i^q(\boldsymbol{y}). \end{aligned}$$

Similarly, for the case of $\alpha \neq 1$,

$$\begin{aligned} &\frac{\partial \Phi}{\partial y_i^q}(\boldsymbol{y}) \\ &= -\frac{\partial}{\partial y_i^q}\left[\sum_{i\in\mathcal{S}} \frac{(1-\rho_i)^{1-\alpha}-1}{\alpha-1} + \eta\Phi_G(\boldsymbol{y})\right] \\ &= -\left[\frac{1-\alpha}{\alpha-1}\cdot(1-\rho_i)^{-\alpha}\cdot\left(-\frac{\partial \rho_i}{\partial y_i^q}\right) + \eta(1-q_i)P_i\frac{\partial \rho_i}{\partial y_i^q}\right] \\ &= -\varrho_i^q\left[(1-\rho_i)^{-\alpha} + \eta P_i(1-q_i)\right] = F_i^q(\boldsymbol{y}). \end{aligned}$$

■

Lemma 2: The potential function $\Phi(\boldsymbol{y})$ is concave in $\boldsymbol{y}$.

Proof: It has been proved by [10] that Φ is concave in $\boldsymbol{\rho}$. Since Φ is non-increasing in $\boldsymbol{\rho}$, and $\boldsymbol{\rho}$ is concave over $\boldsymbol{y}$ (since $\boldsymbol{\rho}$ is linear combination of the components of $\boldsymbol{y}$). From concavity-preserving operations, the composition of two functions $\Phi(\boldsymbol{\rho})$ and $\boldsymbol{\rho}(\boldsymbol{y})$ becomes $\Phi(\boldsymbol{y})$ which is concave in $\boldsymbol{y}$. ■

From Theorem 1, the NE of our game can be easily characterized by KKT condition of the potential function Φ. Therefore, all NE points satisfy the KKT condition of the potential function Φ. Lemma 2 guarantees that the local maxima (i.e., NEs) are also the global maxima of the potential function Φ. Note that the uniqueness of NE is not guaranteed, which means there can be multiple association scenarios at NEs.

Theorem 2: Our association game defined by the marginal payoff function (5) has zero PoA.

Proof: Lemma 2 implies that the optimization problem (2) is indeed convex optimization problem, and has zero duality gap. Thus, the points satisfying KKT condition of the problem globally maximize the social objective function (2). Also, from Lemma 1, the social objective function is a potential function. Therefore, from Theorem 1, the NE of our game coincides with the point satisfying KKT condition of the potential function. Hence, it is guaranteed that NE actually exists (derived from KKT condition), and all NE points globally maximizes the social objective function. ■

IV. Evolutionary Dynamics

In this section, we consider *evolutionary dynamics* to study how users' association evolves over time and converges (if it does). We consider three popular dynamics in the area of population games, discuss their convergence to NE, and connect them to practical, distributed association algorithms.

An evolutionary dynamic is expressed by a differential equation $\dot{\boldsymbol{y}} = V(\boldsymbol{y})$, where $V : \mathcal{Y} \to \mathbb{R}$ is a state-dependent vector field which defines the drift of the social state. To transfer a dynamic based on a differential equation to a concrete association algorithm, each BS i broadcasts to users (belonging to a class having BS i as available BSs) necessary information (e.g., load), which differs for each dynamic. Each user is equipped with its individual Poisson clock with unit rate and updates its strategy (i.e., association) whenever the clock ticks, following the rule governed by the dynamic. Clock-based strategy updates by users enable the system to operate asynchronously, preventing possible oscillations and ping-pong effects.

A. Replicator and BNN Dynamics

The first dynamic widely used in evolutionary dynamics is replicator dynamic. Its basic idea is to form a drift vector based on the average payoff of the corresponding class, where the drift is made, so that each user prefers a strategy with larger *excess payoff* (i.e., the difference between the current strategy's payoff and the average payoff). The replicator dynamic is described as:

$$\dot{y_i^q} = V(\boldsymbol{y}) = y_i^q \left(F_i^q(\boldsymbol{y}) - \frac{1}{d^q} \sum_{i \in \mathcal{S}^q} y_i^q F_i^q(\boldsymbol{y}) \right), \tag{6}$$

where the term $F_i^q(\boldsymbol{y}) - \frac{1}{d^q} \sum_{i \in \mathcal{S}^q} y_i^q F_i^q(\boldsymbol{y})$ corresponds to the excess payoff of the strategy i in class q. The replicator dynamic is an instance of *imitative protocols*. In other words, at each update epoch, each user in the class randomly encounters another user, called opponent. If the payoff of the opponent exceeds the user's own payoff, then the user selects the opponent's strategy with probability proportional to the payoff difference among two encounters. Replicator dynamic captures the strategy popularity as well as the excess payoff of each strategy in the sense that the strategy drift is both proportional to the excess payoff of the strategy and the number of users playing the strategy. In the association algorithm motivated by replicator dynamic, we emulate random encountering by letting the BSs distribute required information to the users, as discussed later.

The second dynamic is Brown-von Neumann-Nash (BNN) dynamic. For ease of exposition, we first define a variable k_i^q to be the maximum of the excess payoff and zero: $k_i^q \triangleq \max\left\{F_i^q(\boldsymbol{y}) - \frac{1}{d^q} \sum_{i \in \mathcal{S}^q} y_i^q F_i^q(\boldsymbol{y}), 0\right\}$. Then, BNN dynamic is expressed as:

$$\dot{y_i^q} = V(\boldsymbol{y}) = d^q k_i^q - y_i^q \sum_{i \in \mathcal{S}^q} k_i^q. \tag{7}$$

The intuition behind BNN dynamic is that at each update epoch, each user randomly picks a strategy and compares its payoff with the average payoff. If the payoff of the chosen strategy exceeds the average payoff, the user changes its own strategy with probability proportional to the excess payoff.

We now describe the association algorithms motivated from two dynamics. We assume that all users know the system parameters P_i, η, q_i, and α.

Two Association Algorithms from Replicator and BNN

- Each BS i: At each iteration time slot, it broadcasts ρ_i and y_i^q to the users belonging to a class in $\mathcal{Q}(i)$.
- Each user u in each class q: Whenever its clock ticks, the user calculates the marginal payoffs $(F_k^q : k \in \mathcal{S}^q)$, using the most recent BS-broadcasted information of $(\rho_i : i \in \mathcal{S}^q)$ and $\boldsymbol{y}^q = (y_i^q : i \in \mathcal{S}^q)$. Let j be the BS that the user u is currently associated with.
  - ***Replicator:*** The user computes the payoff differences for all other BSs: $D_{ji}^q \triangleq \max(F_i^q - F_j^q, 0)$ for each BS $i \in \mathcal{S}^q$. Then, the user randomly selects a new BS j' with probability proportional to $y_{j'}^q$ and switches its association to BS j' with probability $D_{j'j}^q / \sum_{i \in \mathcal{S}^q} D_{ij}^q$.
  - ***BNN:*** The user computes the average payoff $\bar{F}^q \triangleq \sum_{i \in \mathcal{S}^q} y_i^q F_i^q / d^q$. The user computes the payoff differences for all other BSs: $D_i^q \triangleq \max(F_i^q - \bar{F}^q, 0)$ for each BS $i \in \mathcal{S}^q$. Then, the user randomly selects new BS j' uniformly, and switches to new BS j' with probability $D_{j'}^q / \sum_{i \in \mathcal{S}^q} D_i^q$.

B. Best Response (BR) Dynamic

In the BR dynamic, at a given social state, each user attempts to select its strategy that gives a maximum payoff. Mathematically, BR dynamic is expressed by the map from each state to the differential inclusion:

$$\dot{y^q} \in V^q(\boldsymbol{y}) = B^q(\boldsymbol{y}) - y^q, \tag{8}$$

where recall that $B^q(\boldsymbol{y})$ is the mixed best response correspondence of the social state $\boldsymbol{y}$. The BR-based association algorithm is described as follows:

Association Algorithm from BR

- Each BS i: At each iteration time slot, it broadcasts ρ_i to the users belonging to a class in $\mathcal{Q}(i)$.
- Each user in class q: Whenever its clock ticks, using the most recent broadcasted information of $(\rho_i : i \in \mathcal{S}^q)$, the user calculates the marginal payoffs $(F_k^q : k \in \mathcal{S}^q)$. Then, it selects the new BS that provides the maximum marginal payoff, i.e., the users select the BS j^q satisfying:

$$j^q = \arg\max_{i \in \mathcal{S}^q} \left(-\varrho_i^q \left[(1-\rho_i)^{-\alpha} + \eta(1-q_i)P_i \right] \right). \quad (9)$$

The algorithms motivated by replicator and BNN dynamics involve the emulation of random encountering and the comparison with the average payoff. Both procedures require the information of the strategic distributions of class which an user belongs to. In association algorithms, a BS broadcasts the strategic distribution vector $\boldsymbol{y}^q$ to the users in the class q. However, the BR dynamic does not need such strategic distributions, and the algorithm only compares the payoff values across various strategies (i.e., BSs) and the payoff value can be calculated by using only BS load vector ρ.

In [9], [10], the authors proposed a distributed algorithm that solves the social optimization problem in (2) from an optimization-theoretic perspective, described as: at each iteration k,

$$j^{(q,k)} = \arg\max_{i \in \mathcal{S}^q} \frac{c_i^q}{(1-\rho_i^{(k)})^{-\alpha} + \eta(1-q_i)P_i}. \quad (10)$$

We can easily check that using $c_j^q = \gamma^q / \varrho_j^q$, the criterion of selecting BS for both algorithms—(9) for the game-inspired algorithm from the best response dynamic and (10) for the distributed algorithm in [10]—are equivalent. However, there is remarkable difference in implementing two algorithms. In the algorithm proposed in [9], [10], all users implicitly reconsider their BS selection strategy synchronously when BSs broadcast their loads. Our BR dynamic assigns individual Poisson clock to each user and runs asynchronously. This subtlety may incur huge performance difference when the number of classes is finite and thus when the cell boundary is not a line but a region (so its measure is non-zero); indeed a practical data rate setup using AMC instead of continuous Shannon capacity falls into this category. Under this circumstance, the deterministic user association [9], [10] forces all users in the cell boundary to select the same BS resulting in *lumped* user association. However, the optimal user association should be performed in a *probabilistic* way. Our BR dynamic successfully splits users in the cell boundary with the optimal ratio. Consequently, even though BR dynamic is *individually deterministic* it behaves in a *collectively probabilistic* way and converges to the optimal point.

C. Convergence and Comparison

This section is devoted to summarizing the convergence of three dynamics and discussing their differences in convergence speed inside our association problem context. Convergence of three dynamics has been well studied in literature. Thus we refer the readers to [13], [24], [25] for more details. We first define a notion of *positive correlation (PC)* related to a sufficient condition under which an evolutionary dynamic converges to NE.

Definition 3: $\dot{\boldsymbol{y}} = V(\boldsymbol{y})$ is positively correlated if

$$V(\boldsymbol{y}) \cdot F(\boldsymbol{y}) = \sum_{q \in \mathcal{Q}} \sum_{i \in \mathcal{S}^q} F_i^q(\boldsymbol{y}) V_i^q(\boldsymbol{y}) > 0 \text{ whenever } V(\boldsymbol{y}) \neq 0.$$

Positive correlation states that the drift rate and the payoff values are positively correlated. In potential games, if the dynamic satisfies PC then the potential function becomes *Lyapunov function*; the potential function Φ acts as a (global) Lyapunov function of the dynamic, since for all solution trajectories $\boldsymbol{y}_t$, (i) $\frac{d}{dt}\Phi(\boldsymbol{y}_t) = \nabla\Phi(\boldsymbol{y}) \cdot \dot{\boldsymbol{y}}_t = F(\boldsymbol{y}_t) \cdot V(\boldsymbol{y}_t) \geq 0$ and (ii) $V(\boldsymbol{y}_t) = 0$ whenever $\frac{d}{dt}\Phi(\boldsymbol{y}_t) = 0$ from PC. This means that all solution trajectories of the dynamic satisfying PC are nondecreasing until a stationary point, i.e., a point y with $\dot{\boldsymbol{y}} = V(\boldsymbol{y}) = 0$. Thus, all solution trajectories eventually converge to a stationary point.

All three dynamics are known to be provably positive correlated. However, all stationary points are not necessarily NEs, where the dynamic converges to either *(i)* a local maximum of the Lyapunov function or *(ii)* a boundary point of the set $\mathcal{Y}$. Another condition that enables a stationary point to be a NE is so-called *non-complacency* (NC) or *Nash stationarity*. The BNN and BR dynamics satisfy NC, allowing those two dynamics to converge to a NE. However, the replicator dynamic does not satisfy NC, opening possibility of convergence to a stationary point that is not NE. Nonetheless, as shown in Section V, replicator dynamic seems to converge to NE in many cases. From our experience when there exists a positive portion of players associating with each BS in the initial condition, even the replicator dynamic converges to NE.

We now discuss the convergence speed of three dynamics. The BR dynamic does not perform any probabilistic operations and just switches to the BS providing the largest payoff, whereas the algorithms from replicator and BNN dynamics switch to better BSs probabilistically. This makes the convergence speed of BR faster than that of the other two. However, there is more chance of instability in the BR-based algorithm and it is usually implemented with certain relaxation parameter. Also, the convergence speed of replicator and BNN dynamics depends on initial conditions, as will be numerically verified in Section V. If the initial distribution is biased to one strategy and the stationary distribution is relatively uniform, the convergence speed of BNN dynamic is faster than that of replicator dynamic. This is because replicator dynamic tends to drift to more popular strategy and it is hard to exit from the initial biased point because the initially dominant strategy is relatively more preferred. However, in the opposite case when initial distribution is relatively uniform and the stationary distribution is biased, replicator dynamic converges faster than

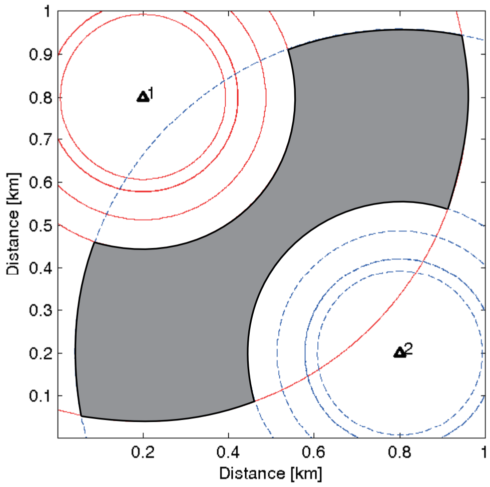


Fig. 2: Representation of simulation environment. The solid and dashed line represent the coverage of BSs 1 and 2, respectively. The shaded region represents the tie region between BSs 1 and 2.

BNN dynamic, because BNN dynamic continuously selects suboptimal strategy uniformly.

V. Simulation Results

We consider a cellular network topology shown in Fig. 2 consisting of two urban macro BSs within 1×1 km^2. The transmission power of each BS is 43dBm, and the maximum operating power of BSs is 865W. SINR value for determining link capacity was calculated from the modified COST 231 Hata path loss model from IEEE 802.16m (mobile WiMAX) document [26]. Based on the calculated SINR values, AMC was also simulated from the mobile WiMAX standard references [27], [28]. The red and blue contours represent the AMC level separations of BS 1 and BS 2, respectively. In this two-cell case, the inter-cell interference term was ignored. This is not unrealistic, because the current cellular standard uses fractional frequency reuse (FFR) and adjacent cells use different frequency band in order to reduce inter-cell interference. Note that the shaded region depicts the potential cell boundary between BS 1 and BS 2; all users in these region receive the same data rate from two BSs, and the decision metric becomes identical (i.e., a tie occurs) when the loads of two BSs are equal.

Fig. 3 shows the simulation results under this setup. The load-balancing factor α is set to 2, i.e., delay-optimal, and the energy-delay tradeoff factor η is set to 10^{-1}. Both BSs are assumed to be energy-proportional. Due to space limitation, we show the plots for the above-mentioned parameters, but we observed similar trends for other choices of system parameters. We assume the spatially homogeneous traffic distribution, so we henceforth denote γ^q as just γ. Homogeneous traffic distribution is just adopted for simplicity, but similar interpretations in this section can be made for other heterogeneous cases.

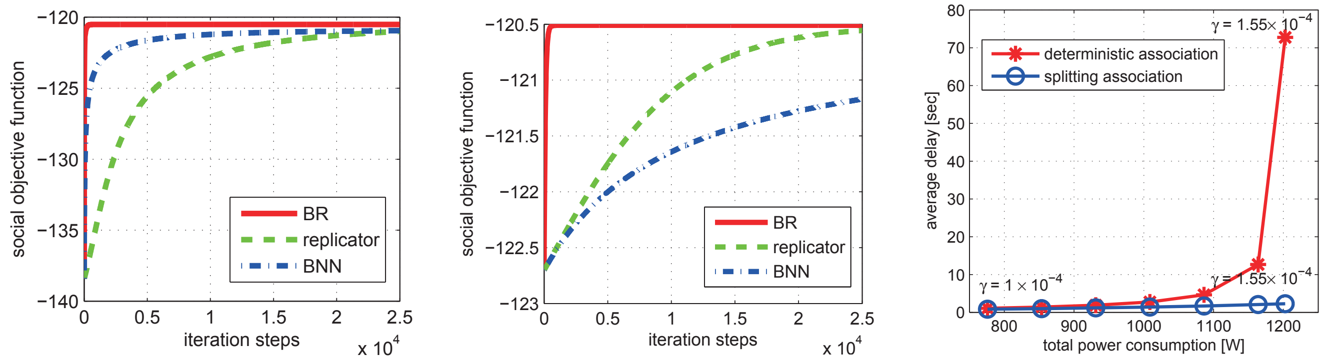
First, Figs. 3(a) and (b) show the time-varying behavior of three dynamics starting from different initial points. In terms of the convergence, three dynamics converge to the same point, which is a NE and also the socially optimal solution, as seen in Fig. 3(a). However, the convergence speed of each dynamic depends on the initial points except the BR dynamic. As seen in Figs. 3(a) and (b), the BR converges fastest, but that replicator and BNN dynamics show situation-dependent convergence speed, as discussed next.

Fig. 3 (a) starts with heavily (1% vs 99%) biased association in the shaded region in Fig. 2. The socially optimal association in this region should be the 50%–50% split distribution due to symmetry. As shown in the Fig. 3 (a), replicator dynamic exits from the initial point much more slowly than other dynamics. This occurs because replicator dynamic tends to select more "popular" BSs. Fig. 3 (b) shows the opposite scenario. The initial point is set as equally split distribution in deciding BSs except one of the non-shaded region in Fig. 2. In non-shaded region, the optimal strategy is that all users simply select the BS that gives the higher data rate. Starting from 50%–50% distribution, replicator dynamic converges fast to the optimal distribution, whereas BNN dynamic does not. The intuition is as follows; while in replicator dynamic users tend to pick more popular strategies, in BNN dynamic users choose one strategy at random and compare its payoff with average payoff at each selection instant. In BNN dynamic, users are more likely to make choose suboptimal decisions, compared to replicator dynamic, resulting in drastically slow convergence. Note again that BR dynamic needs not worry about the initial point and dominates other two dynamics in convergence speed.

Finally, Fig. 3 (c) shows the remarkable difference between [9], [10] and our work. We see an order of magnitude difference in delay for the same power consumption when the load is high. As explained in Section IV-B, the users in the cell boundary make the same decision in [9], [10]. Hence, the algorithm in [9], [10] cannot avoid ping-pong effect happening in the shaded region under the setup in this paper. However, the proposed dynamics including BR dynamic achieves an *optimally* split association. Simulations were performed for varying traffic density γ. As γ increases, the total power consumption increases. Since symmetric BS setting is assumed, total power consumption is the same in both deterministic and splitting scenarios. As shown in Fig. 3 (c), the split association scheme is far more efficient than the one in [10] in terms of minimizing delay. Specifically when $\gamma = 1.55 \times 10^{-4}$, the deterministic association generates heavily unbalanced load between two BSs and the average flow delay becomes almost 32 times larger than our splitting association. We then can conclude that the splitting association plays an important role for load balancing when traffic density γ is high and becomes more practical than the one in [9], [10].

VI. Conclusion

In this paper, we have studied an energy-efficient BS association problem from a population game perspective. Our study



(a) Case I: The initial associating distribution in the shaded region in Fig. 2 is 1%-99% biased selection.

(b) Case II: The initial associating distribution in one of the non-shaded region in Fig. 2 is 50%-50%, whereas the optimal distribution is deterministic. The initial associating distribution in the shaded region is 50%-50% distribution, which is optimal.

(c) Growth of average delay and power consumption with increasing traffic density γ.

Fig. 3: Simulation results: convergence and delay vs. power consumption with increasing traffic density

has revealed that the proposed distribution association algorithms (motivated by various evolutionary dynamics) converge to the socially optimal point through appropriate association pricing. Future work will more focus on the behavior at the cell edge and the practical implementation of the algorithms. For example, in the case of BR dynamic, it is of interest to determine how to control the Poisson clock to guarantee fast convergence as well as no (or little if any) oscillation of user association.

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