Green DSL: Energy-Efficient DSM

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***Abstract*—Dynamic spectrum management (DSM) has been recognized as a key technology for tackling multi-user crosstalk interference for DSL broadband access. Up to now, DSM design has mainly been focusing on maximization of data rates. However, recently, reducing the total power has become a main target, as IT power consumption has been identified as a significant contributor to global warming. In this paper we extend traditional DSM design towards a much wider energy-efficient scope and show how to tackle the corresponding optimization problems. The impact of this 'green DSL' approach is evaluated for practice with some surprisingly good numerical results. Furthermore bounds are provided on the trade-off between data rate performance and power saving.**

I. Introduction

Digital subscriber line (DSL) technology refers to a family of technologies that provides digital broadband access over the local telephone network. It is still the dominating broadband access technology with 66% of all broadband access subscribers worldwide using DSL to access the Internet [1]. In order to cope with the increasing demands of the users (end-users as well as service providers) and to stay competitive with other broadband access technologies, DSL technology is continuously extended to encounter the corresponding technological issues. One of the major challenges, is to overcome the electromagnetic interference, also called *crosstalk*, generated among different lines operating in the same cable bundle. Different lines (i.e. users) indeed interfere with each other, leading to a very challenging interference environment where proper management is required to prevent a huge performance degradation.

Dynamic spectrum management (DSM) is recognized as a key technology for tackling this crosstalk problem [2]. The main idea of DSM is to prevent and/or remove crosstalk by using spectrum and/or signal coordination of all users, respectively. We will focus on spectrum coordination, also referred to as spectrum balancing or multi-carrier power control.

The major research efforts in DSM[1] algorithm design have been focusing on maximizing the data rates (i.e. rate-adaptive DSM [3]) without any power minimizing design objective (margin-adaptive [3]), except for the power constraints defined by DSL standards. However recently power consumption has started to gain a lot of importance (ITU-T Study Group 15, European Code of Conduct for broadband equipment). Information and Communication Technologies (ICTs) have been identified as significant contributors to *global warming* [4]. Broadband equipment contributes to the electricity consumption and depending on the penetration level, the specifications of the equipment and the requirements of the service provider, a total European consumption of up to 50 TWh per year can be estimated for the year 2015 [5]. Therefore the European Code of Conduct for Broadband Equipment takes initiative in setting up general principles and actions, and targets to limit the (maximum) electricity consumption to 25 TWh per year which is equivalent to 5.5 Million ton of oil equivalent (TOE) and to a total saving of about € 7.5 Billions per year. DSL, as the most deployed broadband technology, plays an important role in this setup [6]. The DSL Forum encourages international standards bodies to develop techniques for power reduction within the scope of their activities and to maximize the savings while preserving and enhancing quality of service [6]. One technology that fits well in this framework is DSM.

This gave us the motivation to revisit DSM and extend its design with power related objectives. This approach would benefit from the traditional pure data rate maximizing approach in two ways [7]: (i) Adding objectives and/or constraints for limiting transmit power reduces overall power consumption by DSL systems, as power consumed by DSL modems is often dominated by circuits used for transmitting power, and (ii) smaller transmit powers encourage a 'polite' behaviour as less crosstalk is radiated into other DSL modems. These benefits of making the copper 'greener' in an evolving and increasingly energy-efficient world, were recently also pointed out by the authors in [7].

The main contributions of this paper are as follows:

(i) We extend the traditional rate-adaptive DSM design to a much wider 'green' setting, incorporating power limiting objectives and/or constraints. This unifying 'green DSL' framework leads to a much larger freedom and potential of DSM in managing QoS for DSL broadband access.

(ii) We provide a systematic procedure for tackling this extended set of (non-convex) DSM problem formulations by introducing extended Lagrange multipliers and slightly modifying existing DSM algorithms.

(iii) We demonstrate the substantial potential of 'green DSL' in achieving large power savings of up to 50% while still achieving 85% of full-power data rate performance for realistic DSL scenarios. Furthermore lower bounds are provided on the data rate performance versus power saving.

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[1]In this text, DSM refers to spectrum coordination.

II. System Model

We consider a system model consisting of N interfering (synchronous) DMT-DSL modems or users. We denote by K the number of frequency bands or tones available for each user. We abuse the notations N and K to refer to the index set of users and tones. Modems employ single-user encoding and decoding (treating interference as noise). This results in the following characterization of the achievable rate region $\mathcal{R}$:

$$\mathcal{R} = \Big\{(R^n : n \in N) | R^n = f_s \sum_{k \in K} b_k^n(\mathbf{s}_k), \mathbf{s} \in \mathcal{S}\Big\},$$

where

$$\mathcal{S} = \Big\{\mathbf{s} \in \mathbf{R}^{N \times K} : s_k^n = [\mathbf{s}]_{n,k}, \textstyle\sum_{k \in K} s_k^n = P^n, \quad P^n \leq P^{n,\text{tot}}, 0 \leq s_k^n \leq s_k^{n,\text{mask}}\Big\}, \tag{1}$$

$$b_k^n(\mathbf{s}_k) \triangleq \log_2\left(1 + \frac{1}{\Gamma}\frac{|h_k^{n,n}|^2 s_k^n}{\sum_{m \neq n} |h_k^{n,m}|^2 s_k^m + \sigma_k^n}\right) \text{bits/Hz}, \tag{2}$$

and where R^n denotes the data rate of user n, f_s denotes the DMT symbol rate, s_k^n denotes the transmit power of user n on tone k, b_k^n, which depends on the transmit powers $\mathbf{s}_k = [s_k^1, \ldots, s_k^N]^T$, denotes the bit rate of user n on tone k, $P^{n,\text{tot}}$ denotes the total power budget available to user n and $s_k^{n,\text{mask}}$ denotes the spectral mask constraint for user n on tone k. Furthermore $[\mathbf{H}_k]_{n,m} = h_k^{n,m}$ is an $N \times N$ matrix containing the channel transfer functions from transmitter m to receiver n on tone k. The diagonal elements are the direct channels, the off-diagonal elements are the crosstalk channels. σ_k^n denotes the noise power on tone k in receiver n that contains thermal noise, alien crosstalk and radio frequency interference (RFI). Γ denotes the signal-to-noise ratio (SNR) gap to capacity, which is a function of the desired bit error ratio (BER), the coding gain and noise margin [8]. Note that set $\mathcal{S}$ constitutes the set of constraints that are imposed by DSL standards, including total power constraints as well as spectral mask constraints.

III. Green DSL

A. Unifying Green DSL Formulation

The most common approach [9]–[14] to DSM design is a fully data rate driven approach where a weighted sum of data rates is to be optimized as follows:

$$\begin{aligned} &\max_{\{s_k^n, k \in K, n \in N\}} \sum_{n \in N} w_n R^n \\ &\text{s.t. } P^n \leq P^{n,\text{tot}}, n \in N \\ &\quad 0 \leq s_k^n \leq s_k^{n,\text{mask}}, k \in K, n \in N \end{aligned} \tag{3}$$

where $w_n, \forall n$, are weights to specify the importance of each user n. Extra lower bounds on the data rates $R^n \geq R^{n,\text{target}}, \forall n$, are sometimes also added, where $R^{n,\text{target}}$ corresponds to the minimum data rate requirement for user n.

A first relevant extension towards an energy-minimizing approach would be to impose a constraint on the sum of all allocated transmit powers so as to reduce the total consumed power by factor α, leading to the following DSM design:

$$\begin{aligned} &\max_{\{s_k^n, k \in K, n \in N\}} \sum_{n \in N} w_n R^n \\ &\text{s.t. } P^n \leq P^{n,\text{tot}}, n \in N \\ &\quad 0 \leq s_k^n \leq s_k^{n,\text{mask}}, k \in K, n \in N \\ &\quad R^n \geq R^{n,\text{target}}, \forall n \in N \\ &\quad \sum_{n \in N} P^n \leq \alpha \sum_{n \in N} P^{n,\text{tot}}, \end{aligned} \tag{4}$$

where α is a chosen constant smaller than 1, and denotes the required power reduction with respect to full power usage.

Another relevant formulation could be to drive the full objective towards energy minimization subject to minimum data rate constraints as follows:

$$\begin{aligned} &\min_{\{s_k^n, k \in K, n \in N\}} \sum_{n \in N} P^n \\ &\text{s.t. } P^n \leq P^{n,\text{tot}}, n \in N \\ &\quad 0 \leq s_k^n \leq s_k^{n,\text{mask}}, k \in K, n \in N \\ &\quad R^n \geq R^{n,\text{target}}, \forall n \in N \end{aligned} \tag{5}$$

A general green DSL formulation can be obtained by introducing both data rates and powers into the objective as follows:

$$\begin{aligned} &\max_{\{s_k^n, k \in K, n \in N\}} \sum_{n \in N} w_n R^n - \sum_{n \in N} t_n P^n \\ &\text{s.t. } P^n \leq P^{n,\text{tot}}, n \in N \\ &\quad 0 \leq s_k^n \leq s_k^{n,\text{mask}}, k \in K, n \in N \\ &\quad R^n \geq R^{n,\text{target}}, \forall n \in N \\ &\quad \sum_{n \in N} P^n \leq \alpha \sum_{n \in N} P^{n,\text{tot}}, \end{aligned} \tag{6}$$

Starting from general problem formulation (6), we can derive a suite of DSM problem formulations, including (4) and (5), by removing constraints and/or parts of the objective function. This leads to the tree structure of Figure 1 where the nodes correspond to DSM problem formulations and the arrows indicate a removal of a constraint or part of the objective function. More specifically, **a** and **b** indicate the presence in the objective of a weighted sum of data rates $\sum_{n \in N} w_n R^n$ and weighted sum of powers $-\sum_{n \in N} t_n P^n$, respectively. **c** and **d** indicate the presence of target data rate constraints $R^n \geq R^{n,\text{target}}$ and global total power constraints $\sum_{n \in N} P^n \leq \alpha \sum_{n \in N} P^{n,\text{tot}}$, respectively. As an example, problem formulations (3), (4), (5) and (6) correspond to nodes **a**, **acd**, **bc** and **abcd** respectively.

All the problem formulations in the tree are non-convex optimization problems because of the non-convex relation between the data rates R^n and the transmit powers s_k^n. Note that only the bold nodes correspond to DSM problem formulations that have been studied up to now. The different formulations in the tree can be interesting for different practical scenarios and this leads to an increased DSM configuration potential where it is the task of the service provider to choose the good formulation to satisfy its corresponding QoS requirements.

B. General Systematic Solution Procedure

In this section we will propose a procedure to tackle the general non-convex optimization problem (6), i.e. node **abcd** in Figure 1, and show that the other formulations can be tackled similarly by redefining the Lagrange multipliers.

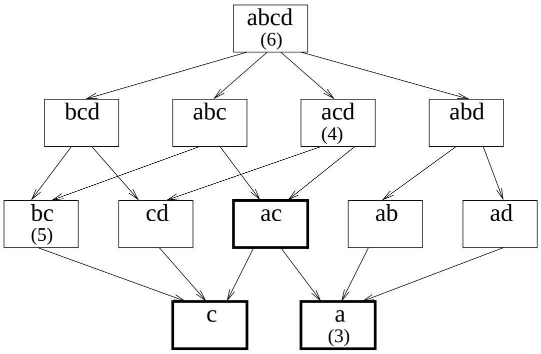


Fig. 1. Tree structure of DSM problem formulations

The procedure uses a *dual decomposition* approach similar to [15], where it was proved that the duality gap for this type of problem formulations can be assumed to be zero (as the number of tones is very large for typical DSL scenarios). More specifically, the procedure consists of solving two main problems, corresponding to a master problem (7) and a slave problem (8), namely:

$$\begin{array}{l} \min_{\theta} L(\theta) \\ \text{s.t. } \theta \geq 0 \end{array} \tag{7}$$

where $\theta = [\nu_1, \ldots, \nu_N, \lambda_1, \ldots, \lambda_N, \lambda_a]^T$, '$\geq$' corresponds to component-wise inequality and the dual function $L(\theta)$ corresponds to the slave optimization problem, which can be decomposed in K independent subproblems as follows:

$$\begin{array}{l} L(\theta) = \sum_k L_k(\theta) \\ \text{with } L_k(\theta) = \left\{ \begin{array}{l} \max_{\mathbf{s}_k} \sum_n \tilde{w}_n f_s b_k^n - \sum_n \tilde{\lambda}_n s_k^n \\ \text{s.t. } 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \quad \forall n \in \mathcal{N}, \end{array} \right. \end{array} \tag{8}$$

where $\tilde{w}_n = \omega_n + \nu_n$, $\tilde{\lambda}_n = t_n + \lambda_n + \lambda_\alpha$ and where Lagrange multipliers $\nu_n, \lambda_n, \lambda_a$ correspond to constraints $R^n \geq R^{n,\text{target}}, P^n \leq P^{n,\text{tot}}$ and $\sum_n P^n \leq \alpha \sum_n P^{n,\text{tot}}$ respectively. Note that (8) is similar to the slave problems of the pure data rate driven DSM formulations, e.g. (3), where the Lagrange multipliers $\tilde{w}_n, \tilde{\lambda}_n, \forall n$, are defined differently.

The solution of (7), i.e. the optimal Lagrange multipliers so that all constraints are satisfied, can be obtained using the following subgradient updates [15], until the corresponding KKT complementarity conditions are satisfied:

$$\nu_n = [\nu_n + \delta(R^{n,\text{target}} - R^n)]^+, \ \forall n, \tag{9}$$

$$\lambda_n = [\lambda_n + \epsilon(P^n - P^{n,\text{tot}})]^+, \ \forall n, \tag{10}$$

$$\lambda_a = [\lambda_a + \phi(\sum_n P^n - \alpha \sum P^{n,\text{tot}})]^+, \tag{11}$$

where δ, ϵ, ϕ are stepsizes that can chosen using different approaches [15] [16] and where $[x]^+ = \max(x, 0)$.

For given Lagrange multipliers θ, the K slave subproblems (8) can be solved using existing DSM algorithms (IW, ASB(2), SCALE, MIW, DSB, MS-DSB, OSB) by replacing the usual Lagrange multipliers by the extended Lagrange multipliers $\tilde{w}_n, \tilde{\lambda}_n, \forall n$. Each of these algorithms has its own properties trading-off complexity and performance (sub optimal, locally optimal, globally optimal) as explained in [14].

This procedure for problem formulation **abcd** in Figure 1 can also be used for all the other problem formulations in the tree by redefining the extended Lagrange multipliers and/or removing some Lagrange multiplier updates. This is summarized in table I where $\mathcal{B}$ consists of the set of Lagrange multiplier update formulas. Note that for the traditional DSM formulation (3), i.e. node **a**, the extended Lagrange multipliers $\tilde{w}_n, \tilde{\lambda}_n$, correspond to the weights w_n and the usual Lagrange multipliers λ_n, respectively.

TABLE I
DEFINITION OF EXTENDED LAGRANGE MULTIPLIERS $\tilde{w}_n, \tilde{\lambda}_n, \forall n$, FOR DIFFERENT PROBLEM STATEMENTS

Problem	$\tilde{w}_n$	$\tilde{\lambda}_n$	$\mathcal{B}$
abcd	$w_n + \nu_n$	$\lambda_n + t_n + \lambda_a$	(9)(10)(11)
bcd	ν_n	$\lambda_n + t_n + \lambda_a$	(9)(10)(11)
abc	$w_n + \nu_n$	$\lambda_n + t_n$	(9)(10)
acd	$w_n + \nu_n$	$\lambda_n + \lambda_a$	(9)(10)(11)
abd	w_n	$\lambda_n + t_n + \lambda_a$	(10)(11)
bc	ν_n	$\lambda_n + t_n$	(9)(10)
cd	ν_n	$\lambda_n + \lambda_a$	(9)(10)(11)
ac	$w_n + \nu_n$	λ_n	(9)(10)
ab	w_n	$\lambda_n + t_n$	(10)
ad	w_n	$\lambda_n + \lambda_a$	(10)(11)
c	ν_n	λ_n	(9)(10)
a	w_n	λ_n	(10)

IV. PERFORMANCE IMPACT OF GREEN DSL

What is the impact of the proposed energy-minimizing approaches for realistic DSL scenarios in terms of data rate performance and power saving? Is there a bound on the data rate performance loss for a given total power saving? Answering these questions will give us a clear picture of the potential of energy-minimizing DSM in the green DSL setting. For this, we will start from simulations of realistic DSL scenarios and theoretically analyze the obtained results.

The following parameter settings are assumed for the DSL scenarios. The twisted pair lines have a diameter of 0.5 mm (24 AWG). The maximum transmit power is 20.4 dBm. The SNR gap Γ is 12.9 dB, corresponding to a coding gain of 3 dB, a noise margin of 6 dB and a target symbol error probability of 10^{-7}. The tone spacing Δ_f is 4.3125 kHz. The DMT symbol rate f_s is 4 kHz. Note that we use the MS-DSB algorithm [14] with modified Lagrange multipliers as in table I to solve the DSM problems in this section. In [14] it is shown that this algorithm is very effective in achieving globally optimal performance for most practical DSL scenarios.

In order to quantify the performance impact, we introduce the following power efficiency performance measure:

Definition IV.1 (Power usage c)**.** *The power usage c for the unifying green DSL formulation (6) is defined as the ratio of the sum of all allocated transmit powers and the sum of all available power budgets defined by DSL standards, as follows*

$$c(\Omega) \triangleq \frac{\sum_{n \in N} P^{n,*}(\Omega)}{\sum_{n \in N} P^{n,\text{tot}}} \leq \alpha \leq 1, \tag{12}$$

where $P^{n,*}(\Omega)$ refers to the optimal allocated power of user n for (6) with given parameters $\Omega = (w_1, \ldots, w_N, t_1, \ldots, t_N, R^{1,\text{target}}, \ldots, R^{N,\text{target}}, \alpha)$.

One relevant DSL scenario is shown in Figure 2(a). This is a so-called near-far scenario which is known to be challenging, where DSM can make a substantial difference. Its corresponding rate region is shown in Figure 2(c) where the blue curve is the rate region at full power, obtained by solving the traditional DSM formulation (3), and the green curve is the rate region at half power, obtained by solving the proposed DSM formulation (4) with $\alpha = 0.5$. In Figure 2(d) the blue curve shows the percental data rate performance as a function of the power usage c for this near-far scenario. This curve is obtained by solving the proposed DSM formulation (5) for the different target rates $R^{n,\text{target}}, \forall n$, indicated in Figure 2(c) with red circles, corresponding to 40%, 50%, 60%, 70%, 80%, 90%, 95% and 100% of the achievable data rate performance. More formally this curve corresponds to the percental data rate performance β as a function of $c(0, 0, 1, 1, \beta R^{1,o}, \beta R^{2,o}, 1)$, where $(R^{1,o}, R^{2,o})$ is indicated in Figure 2(c) and β ranges from 0 to 1. One can observe that saving 50% of total power leads to only a small decrease in achievable rate region (green curve in Figure 2(c)). More specifically, it corresponds to 85% of data rate performance as can be seen in Figure 2(d) (blue curve).

To further understand this phenomenon, the evolution of the bit loading is shown in Figure 2(e) for a linearly increasing power budget ranging from 20% to 100% (i.e. full power) in steps of 10%. One can observe a law of diminishing returns, i.e. a linear increase of power leads to a less than linear increase in data rate. In other words, the higher the bit rate, the less effective that power becomes.

In Figure 2(b) a symmetric DSL scenario is shown. Its corresponding trade-off between data rate performance and power saving is shown as the red curve in Figure 2(d). One can observe a data rate performance of 72% for 50% of power saving. Furthermore the evolution of the bit loading is shown in Figure 2(f) for a linearly increasing power budget ranging from 10% to 100% in steps of 10%. One can observe here that the effect of diminishing returns is less obvious than for Figure 2(e). The data rate performance for given power usage thus depends on the type of scenario.

The following theorem IV.1 provides a lower bound on the optimal data rate performance for different levels of power saving, irrespective of the scenario.

Theorem IV.1. *In the worst case the optimal data rate performance decreases linearly as a function of a decreasing power usage c. Furthermore, for a minimum bit loading of 1 bit after power reduction, the lower bound on the optimal data rate performance $g_{1\text{bit}}$ as a function of the power usage c is given by following relation*

$$g_{1\text{bit}}(c) = \log_2(1 + 1/c)^{-1}. \tag{13}$$

Proof: A first observation is that the worst case corresponds to a zero-crosstalk case. This is easy to understand if we know that the more crosstalk is present, the more power we need to increase the data rate and so the less effective that power becomes, leading to a smaller slope of the data rate performance as a function of the power usage. For this zero-crosstalk worst case scenario the percental data rate performance in function of the power usage c on a tone for a user can be expressed as

$$f(\text{SNR}, c) = \frac{\log_2(1 + SNR \times c)}{\log_2(1 + SNR)}, \tag{14}$$

where SNR denotes the signal-to-noise ratio. Note that although this focuses on only one tone and user, the global effect can be seen as an average of these effects on all tones and users. The function f is increasing in SNR. Using l'Hôpital's rule, one can verify the following:

$$\begin{array}{l} \lim_{\text{SNR}\to 0} f(\text{SNR}, c) = c \\ \lim_{\text{SNR}\to \infty} f(\text{SNR}, c) = 1 \end{array} \tag{15}$$

This illustrates two points. As the SNR becomes very large, the data rate performance loss will vanish. Secondly, the percental data rate performance is minimized when SNR is zero and this leads to a *linear* absolute lower bound on the optimal data rate performance in function of the power usage c.

However zero SNR has no practical meaning and therefore it is better to put a certain lower bound on the SNR. One practical intuitive way of lower bounding is by enforcing a minimum bit loading after power reduction of one bit so that at least one bit can be transmitted on the line. This corresponds to the constraint $\text{SNR} \times c = 1$, which leads to the following relation:

$$g_{1\text{bit}}(c) = f(1/c, c) = \frac{\log_2(1 + 1)}{\log_2(1 + 1/c)} = \log_2(1 + 1/c)^{-1}. \tag{16}$$

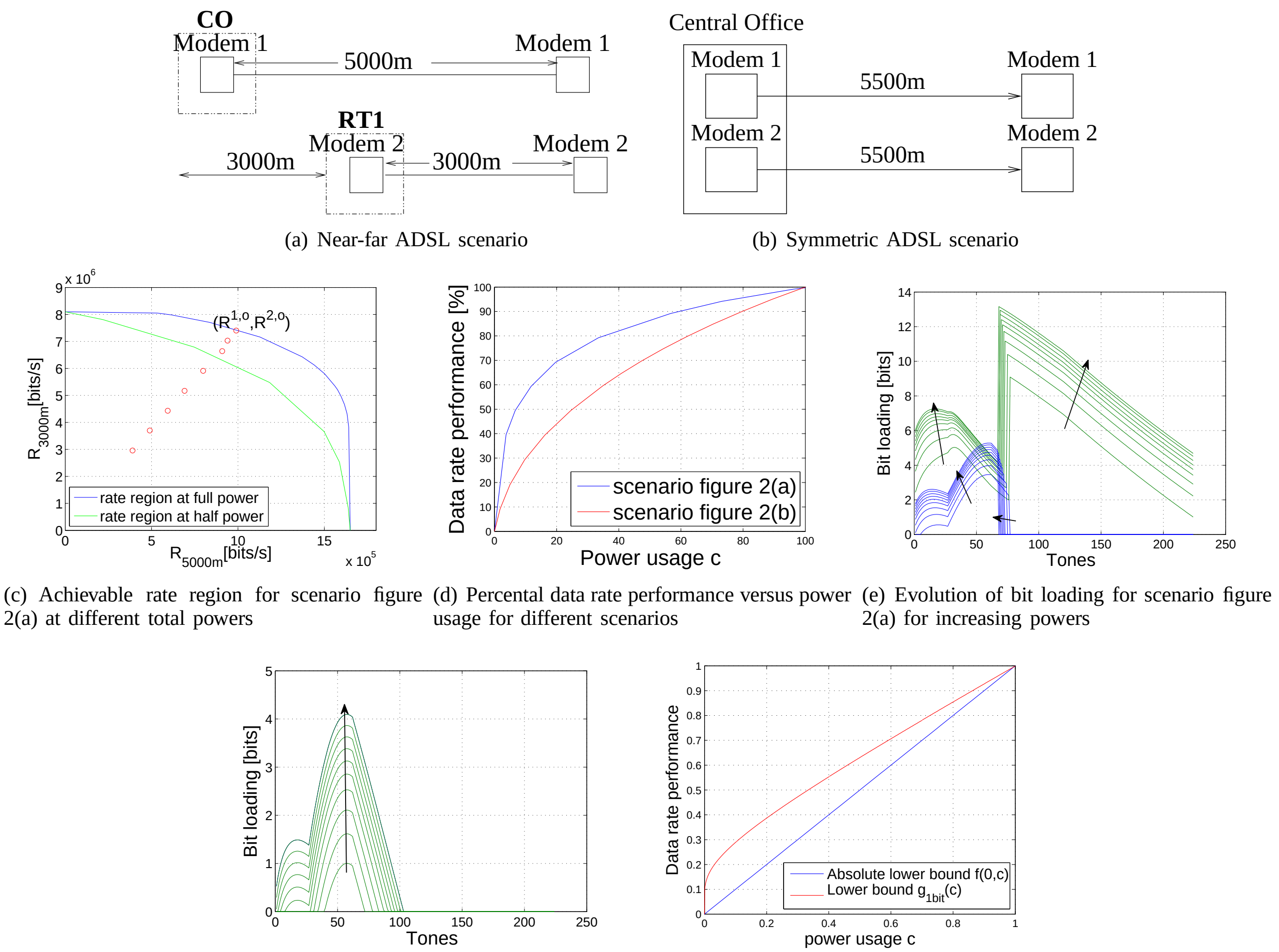
■

Note that (13) corresponds to a lower bound on data rate performance of 63% for 50% power saving. The lower bounds of theorem IV.1 are summarized in Figure 2(g)

This phenomenon of diminishing returns is also quite intuitive as the relation between bits and powers is a logarithm (2). Adding one extra bit requires at least a doubling of the signal-to-noise ratio (SNR), and leads to an increase of the bit rate by a factor $(m + 1)/m$, where m is the bit rate before doubling the power. This means that the higher the bit rate, the less effective power becomes.

In Figure 2(e), the bitloadings range from 0 to 13 bits, with an average of 6 bits. This large average bit rate leads to poor power efficiency. This means that the last added bits require a large amount of power and so by reducing power to 50% of the full power budget, only a small decrease in data rate is incurred, i.e. 85%. In Figure 2(f), the bitloadings range from 0 to 4 bits, with an average of 2 bits. This small average leads to better power efficiency and so also a smaller data rate performance for 50% power usage, i.e. 72%.

Power efficiency depends on the SNR, which depends on the line attenuation, and this in turn depends on the length of the lines. So in scenarios with long line lengths, larger data rate decreases are observed when the total power is reduced. In the limit when SNR goes to zero (very large attenuation or very large noise), the logarithm behaves as a linear function and decreasing power by a factor 2 leads to a decrease in rate of factor 2. However in practice, typical scenarios have an average SNR which is much larger than zero, leading to only small data rate performance losses under large power savings. This result is quite promising for the future of green DSL.



(a) Near-far ADSL scenario

(b) Symmetric ADSL scenario

(c) Achievable rate region for scenario figure 2(a) at different total powers

(d) Percental data rate performance versus power usage for different scenarios

(e) Evolution of bit loading for scenario figure 2(a) for increasing powers

(f) Evolution of bit loading for scenario figure 2(b) for increasing powers

(g) Lower bounds on percental data rate performance vs power usage

Fig. 2. Simulation Scenarios and Results

V. Conclusion

Energy saving is becoming an important goal in broadband access design. Here we provide the first unifying framework for green DSL formulations and provide a systematic procedure for tackling the corresponding optimization problems. Furthermore we evaluate the impact of this energy-efficient approach for realistic DSL scenarios. We show that the worst case corresponds to a linear decrease of the optimal data rate performance for decreasing power usage. In practice, the data rate performance losses are typically much smaller, which is a promising result for this green DSL approach.

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