Practical Dynamic Interference Management in Multi-carrier Multi-cell Wireless Networks: A Reference User Based Approach

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***Abstract*—In order to overcome low performance of conventional static interference management algorithms and high complexity of existing dynamic interference management algorithms, this paper proposes an efficient, low-complex and fully distributed power control and user scheduling algorithm in downlink multi-carrier multi-cell wireless networks. We decompose the original optimization problem, which requires intractable computation complexity for global optimality, into per-BS (base station) problems based on the notion of *reference user*. This idea is a good approximation of the optimal algorithms that permits significant complexity reduction, yet sustains near-optimal performance. We further reduce feedback overhead to make the resulting algorithm more practical by exploiting temporal correlations and spatial simplification. We verify the efficiency of our proposed algorithm through extensive simulations over various topologies and scenarios motivated by the current 3G BS deployment map.**

I. Introduction

Pushed by the demand for bandwidth-hungry multimedia and internet-related wireless services in next-generation wireless cellular networks, communication engineers seek to maximally exploit the spectral resources in all available dimensions. They not only adopt advanced physical layer techniques but also consider the reuse factor close to one with the dense deployment of BSs. In such networks with high reuse, users at cell edges would suffer from low throughput due to severe inter-cell interference (ICI) and unbalanced user distributions among cells. Moreover, achieving reasonable performance of users at cell edges becomes more demanding in heterogeneous cell structures, e.g., coexistence of micro/pico/femto cells with macro cells, because the portion of users whose capacity is limited by ICI grows. Consequently, ICI emerges as the key factor for good performance and the coordination of BSs to effectively manage ICI is essential to fully obtain flexibility and potential of wireless cellular systems.

In managing the ICI, a traditional frequency reuse with a reuse factor greater than one was used in early years of wireless system for voice services. This is a brute-force approach that adjusts the reuse distance to as small as possible while guaranteeing the worst signal quality of the cell-edge users above an acceptable level. More enhanced work such as fractional frequency reuse [1], its variation [2] and soft frequency reuse [3] that allow users in different channel conditions to enjoy different reuse patterns have been proposed. Still, all of these schemes mentioned above are static interference management approaches, where a specific reuse pattern is predetermined a priori by a network operator at offline.

However, in real systems, BSs are not uniformly deployed over the network. Moreover, the distribution of users tends to vary over time. Under this situation, determining the fixed reuse pattern with static scheme becomes far from optimal. Thus, it is necessary that interference management algorithms should be designed to adapt to dynamic network environments, e.g., user load distribution, time-varying channel and interference conditions. Recently, several dynamic interference management algorithms have been proposed to address this problem. They can significantly improve the performance over static interference management algorithms, but many of them suffer from prohibitively high complexity. The fundamental reasons for the high complexity of dynamic schemes are that (i) power allocation problem itself is a highly nonconvex optimization problem[1] and (ii) it is coupled with multi-user scheduling for optimal ICI management, which consists of a multiple of NP-hard problems. In this paper, in order to overcome low performance of conventional static interference management and high complexity of dynamic interference management, we aim at developing a distributed power control and multi-user scheduling algorithm that has low complexity, yet achieves high efficiency.

A. Related Work

There have been a number of works dealing with power allocation in wireless downlink cellular networks. In [4]–[8], power allocation problems are treated in a multi-cell network but with a single resource, i.e., a single-carrier multi-cell network. Under binary power control (BPC) assumption that each BS transmits data with its given maximum power or zero,

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[1]It is a highly nonconvex because system objective metrics (e.g. network utility or weighted sum rate) are eventually nonlinear functions of SINR, which is in turn tightly coupled by transmit powers of all BSs.

several BS coordination schemes finding a set of active BSs have been proposed. The power allocation problem has been handled in the context of uplink cellular or ad-hoc networks as well [9], [10]. Chiang et al. [9] show that in high SINR (signal to interference plus noise ratio) regime nonconvex power control optimization problems can be transformed into convex optimization problems through geometric programming technique.

Other relevant dynamic spectrum management (DSM) algorithms are found in the context of multi-tone DSL (Digital Subscriber Line) networks such as IWF (Iterative Water-filling) [11], ASB (Autonomous Spectrum Balancing) [12], MIWF (Modified Iterative Water-filling) [13], SCALE (Successive Convex Approximation for Low complExity) [14], ISB [15] and OSB [16] (Iterative/Optimal Spectrum Balancing) in increasing order of performance and complexity. IWF is a fully autonomous algorithm where each line tries to selfishly maximize its own data rate by water-filling, but it is highly suboptimal in the asymmetric near-far scenario. ASB is an autonomous algorithm based on the concept of reference line and achieves better performance than IWF. A semi-distributed DSM algorithm called MIWF and SCALE have been also proposed. ISB and OSB are optimal but very complex centralized DSM algorithms, which requires a centralized spectrum management node.

It is worthwhile mentioning clearly what similarities and differences are between the wired multi-tone DSL model and the wireless multi-carrier multi-cell model. Indeed, the wired multi-tone DSL model with crosstalk can be interpreted as the special case of the wireless multi-carrier multi-cell model with ICI when (i) only one user exists per cell and (ii) wireless channels are stationary. Thus, while only the power allocation needs to be solved in the DSL model, the user scheduling reflecting time-varying channel conditions for multiple users should be jointly solved with the power allocation in the multi-cell model. More specifically, in the case of ASB that motivates us to use the reference concept, the reference line that receives the largest cross-talk from each line can be predetermined at offline. However, since scheduled users change slot-by-slot in the multi-cell model, a reference user selection method running at online is required, which seems to be significantly challenging.

In this paper, we consider a multi-carrier multi-cell network where each cell has multiple users, and tackle the power allocation problem along with the user scheduling. Our work differs from the previous work in what follows: (i) The power allocation problem is not treated in the multi-carrier multi-cell domain but in the reduced domain in terms of number of cells and/or carriers [4]–[8], [11]–[16]. (ii) The user scheduling issue is not explicitly considered, i.e., the user selection is predetermined and no joint optimization is undertaken across multiple slots [7]–[16].

There are a few recent works in the same domain as ours [17], [18]. Venturino et al. [17] proposed several efficient near-optimal algorithms with a different level of BS coordination. However, all of them are centralized algorithms that require multiple iteration loops in a slot for user scheduling and power allocation, which hinders a practical implementation. Stolyar et al. [18] proposed algorithms that adjust power allocation much slower than per-slot basis user scheduling. This time-scale separation does simplify the problem solution and reduce the complexity, but may lead to the corresponding performance loss. We will compare these algorithms with ours later in terms of both performance and complexity.

B. Main Contributions and Organization

The main contributions of this paper are as follows.

1) We develop a low-complexity joint power control and user scheduling algorithm that is a fully distributed algorithm requiring only minimal exchange of information between neighboring BSs. Our key idea is an approximation procedure that decomposes a complex original problem into multiple per-BS problems based on the *reference user* concept. Here, the reference user refers to the worst user receiving the largest interference from each BS. We reduce complexity by executing user scheduling and power allocation step-by-step without a loop, which can be done very fast in a slot.
2) Due to the nonconvexity of power allocation problem, a different initial power setting may lead to a different solution with a different speed. We empirically conclude that using the power allocation at the previous slot as an initial power that exploits temporal correlations is a good strategy. It in turn acts as an important component to avoid multiple loops for power allocation and user scheduling (thus significantly reduce the complexity), yet achieve near-optimal performance gain.
3) Our work is general enough to be applicable to heterogeneous BS deployments such as macro, micro, pico and even femto cells because we do not assume the transmit power of BSs are the same. We verify the system performance through extensive simulations under various topologies/scenarios including a conventional hexagonal cell topology, a real 3G BS topology deployed by a major service provider in Korea and a heterogeneous network topology with small cells inside a macro cell. We also investigate computational and signaling complexity.

The remainder of this paper is organized as follows. In Section II, we present our system model. In Section III, we propose a reference user based power allocation and user scheduling. In Section IV, we demonstrate the performance of the proposed algorithm compared to previous algorithms under various topologies and scenarios. Finally, we conclude the paper in Section V.

II. System Model

A. Network and Traffic Model

We consider a wireless cellular network consisting of multiple cells. Denote by $\mathcal{N} \doteq \{1, \ldots, N\}$ and $\mathcal{K} \doteq \{1, \ldots, K\}$ the set of BSs and users (or MSs), respectively. BSs and users are equipped with one transmit and one receive antenna, respectively. Each user is assumed to be connected to a single BS based on long-term average signal strength. Denote by $\mathcal{K}_n$ the (nonempty) set of users associated with the BS n, i.e.,

$\mathcal{K} = \mathcal{K}_1 \cup \cdots \cup \mathcal{K}_N$ and $\mathcal{K}_n \cap \mathcal{K}_m = \emptyset, n \neq m$. Adjacent BSs can in general exchange information with each other because there is logical connections between them via high-speed wired backbone links (or wireless dedicated channel) directly or through a base station controller. We assume that the system is saturated, i.e., an infinite amount of data exists for each users at its associated BS.

B. Resource and Allocation Model

We consider a subchannel that is a group of subcarriers as the basic unit of resource allocation. Assume that there are S number of subchannels and all BSs can use all the subchannels for data transmission, i.e., universal frequency reuse. Denote by $\mathcal{S} \doteq \{1, \ldots, S\}$ the set of subchannels. We focus on the downlink transmissions in the time-slotted system. At each slot, each BS needs to determine (i) *which user is scheduled on each subchannel* and (ii) *how much power is allocated for each scheduled user on each subchannel*.

User scheduling constraint: In regard to (i), denote by $\boldsymbol{I}_s(t) \doteq [I_s^{k,n}(t) : k \in \mathcal{K}, n \in \mathcal{N}]$ the user scheduling indicator vector, i.e., $I_s^{k,n}(t) = 1$ when BS n schedules its associated user k on subchannel s at slot t, and 0 otherwise. Furthermore, we denote the user scheduled by BS n on subchannel s at slot t by $k(n, s, t)$. Reflecting that at most only one user can be selected in each subchannel for each BS, we should have:

$$\sum_{k \in \mathcal{K}_n} I_s^{k,n}(t) \leq 1, \quad \forall n \in \mathcal{N}, s \in \mathcal{S}. \tag{1}$$

Power constraint: In regard to (ii), denote the transmit power of BS n on subchannel s at slot t by $p_s^n(t)$. The vector containing transmit power of all BSs on subchannel s is $\boldsymbol{p}_s(t) \doteq [p_s^1(t), \cdots p_s^N(t)]^T$. In parallel, the vector containing transmit powers of all subchannels for BS n is $\boldsymbol{p}^n(t) \doteq [p_1^n(t), \cdots p_S^n(t)]^T$. Each BS is assumed to have the total power budget and spectral mask constraints:

$$\sum_{s \in \mathcal{S}} p_s^n(t) \ \leq \ P^{n,max}, \quad \forall n \in \mathcal{N}, \tag{2}$$

$$p_s^n(t) \ \leq \ P_s^{n,mask}, \quad \forall n \in \mathcal{N}, s \in \mathcal{S}. \tag{3}$$

For notational simplicity, the time-slot index (t) is dropped except cases in which it needs to be explicitly indicated.

C. Link Model

We do not consider advanced multiuser detection or interference cancellation, and hence interference from other BSs is treated as noise. We focus on the spectrum level coordination[2], i.e., finding multi-channel power allocation of each BS in order to improve system performance by mitigating ICI. For a given power vector $\boldsymbol{p}_s$, the received SINR for user k from BS n on subchannel s can be written as:

$$\gamma_s^{k,n}(\boldsymbol{p}_s) = \frac{g_s^{k,n} p_s^n}{\sum_{m \neq n} g_s^{k,m} p_s^m + \sigma_s^k}, \tag{4}$$

[2]We can further increase the performance by canceling ICI using signal level coordination, such as CoMP (Coordinated Multi Point Transmission and Reception) in the context of LTE-Advanced. This requires high computation/signaling overheads and some changes of radio specification due to joint signal processing and scheduling from multiple geographically separate points. The signal level coordination is beyond the scope of this paper.

where p_s^n and $g_s^{k,n}$ representing the nonnegative transmit power of BS n on subchannel s and the channel gain between BS n and user k on subchannel s, respectively; σ_s^n is the noise power. The channel gain takes into account the path loss, log-normal shadowing and fast fading and etc. Following the Shannon's formula, the achievable data rate [in bps] for user k on subchannel s is given by:

$$r_s^{k,n}(\boldsymbol{p}_s) = \frac{B}{S} \log_2 \left(1 + \frac{1}{\Gamma} \gamma_s^{k,n}(\boldsymbol{p}_s)\right), \tag{5}$$

where B denotes the system bandwidth, S is the number of subchannels and Γ denotes the SNR gap to capacity which is a function of the desired bit error ration (BER), the coding gain and noise margin. Note that $r_s^{k,n}(\boldsymbol{p}_s)$ is the potential data rate when the user k is scheduled for service by BS n on subchannel s and its actual data rate becomes 0 when other user is scheduled, i.e., $r_s^{k,n}(\boldsymbol{p}_s, \boldsymbol{I}_s) = I_s^{k,n} \cdot r_s^{k,n}(\boldsymbol{p}_s)$. As of now, we use $\Gamma = 1$ and drop B/S for simplicity of notations.

III. Reference User Based Power Allocation and User Scheduling

A. General Problem Statement

In this paper, we aim at jointly determining power allocation $\boldsymbol{p} \doteq (\boldsymbol{p}_s, s \in \mathcal{S})$ and user scheduling $\boldsymbol{I} \doteq (\boldsymbol{I}_s, s \in \mathcal{S})$ according to the above resource allocation constraints in multi-carrier multi-cell wireless networks that maximize the long-term utility, i.e., solve the following optimization problem:

$$(\textbf{Long-term P}): \quad \max \quad \sum_{k \in K} U_k(R_k) \tag{6}$$

$$\text{subsect to} \quad \mathbf{R} \in \mathcal{R}, \tag{7}$$

where $\mathbf{R} = (R_k : k \in \mathcal{K})$ is the vector of long-term user throughputs. Assume the standard condition of continuously differentiability and strictly increasing concavity of $U_k(\cdot)$. The set $\mathcal{R} \in \mathbb{R}_+^K$, the set of all achievable rate vectors over long-term, is shown to be a closed bounded convex set.

With the help of the stochastic gradient-based technique in [19] that selects the achievable rate vector maximizing the sum of weighted rates where the weights are marginal utilities at each slot, it suffices to solve the following slot-by-slot problem which produces the long-term rates that is the optimal solution of the (**Long-term P**).

$$(\boldsymbol{P}): \quad \max_{\boldsymbol{p}, \boldsymbol{I}} \quad h(\boldsymbol{p}, \boldsymbol{I}) = \sum_{k \in \mathcal{K}} w_k \sum_{s \in \mathcal{S}} r_s^{k,n}(\boldsymbol{p}_s, \boldsymbol{I}_s) \tag{8}$$

$$\text{subject to} \quad \sum_{k \in \mathcal{K}_n} I_s^{k,n} \leq 1, \qquad \forall n \in \mathcal{N}, s \in \mathcal{S}, \tag{9}$$

$$\sum_{s \in \mathcal{S}} p_s^n \leq P^{n,max}, \quad \forall n \in \mathcal{N}, \tag{10}$$

$$p_s^n \leq P_s^{n,mask}, \qquad \forall n \in \mathcal{N}, s \in \mathcal{S}, \tag{11}$$

where a weight $w_k > 0$ is the derivative of its utility $w_k = \frac{dU(R_k)}{dR_k}|_{R_k = R_k(t)}$ corresponding to the relative priority of user k. For example, we can set the weight of user as the inverse of its average throughput $1/R_k(t)$ to achieve proportional fairness among users [20].

Since the scheduling indicators are integer variables and the system objective is tightly coupled by the transmit powers of all BSs and nonlinear (neither convex nor concave), the problem $(\boldsymbol{P})$ is a nonconvex mixed-integer nonlinear programming (MINLP). Unfortunately, it is known in [16] that even the reduced problem in which user scheduling issue is eliminated (i.e., $|\mathcal{K}_n| = 1$ for all $n \in \mathcal{N}$) is computationally intractable. To find a global optimal solution, we need to fully search the space of the feasible powers for all BSs with a small granularity along with the all possible combinations of user scheduling. Thus, even for a centralized algorithm, it may not be feasible in practical systems having real-time constraint. The objective of this paper is to propose a low-complexity distributed algorithm to efficiently obtain a near-optimal solution.

B. Joint Power Allocation and User Scheduling

Now we develop an efficient algorithm to solve the problem $(\boldsymbol{P})$. Note first that for any given feasible power allocation, the original problem can be decomposed into intra-cell user scheduling problems.

Lemma 3.1: For any fixed feasible power allocating $\boldsymbol{p}$, the original problem $(\boldsymbol{P})$ can be decomposed into $N \times S$ independent subproblems for each BS n and subchannel s as follows:

$$\max_{\boldsymbol{I}_s} \quad \sum_{k \in \mathcal{K}_n} w_k \cdot I_s^{k,n} \cdot r_s^{k,n}(\boldsymbol{p}_s) \tag{12}$$

$$\text{subject to} \quad \sum_{k \in \mathcal{K}_n} I_s^{k,n} \leq 1. \tag{13}$$

Proof: For the given power allocating $\boldsymbol{p}$, we can rewrite $h(\boldsymbol{p}, \boldsymbol{I})$ as follows:

$$\begin{aligned} h(\boldsymbol{p}, \boldsymbol{I}) &= \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}_n} w_k \sum_{s \in \mathcal{S}} I_s^{k,n} \cdot r_s^{k,n}(\boldsymbol{p}_s) && (14) \\ &= \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}} \Big[\sum_{k \in \mathcal{K}_n} w_k \cdot I_s^{k,n} \cdot r_s^{k,n}(\boldsymbol{p}_s) \Big]. && (15) \end{aligned}$$

As w_k and $r_s^{k,n}(\boldsymbol{p}_s)$ are given parameters, we only have to investigate dependencies among $I_s^{k,n}$. Since the constraints (9) on $I_s^{k,n}$ do not play a role across different BSs and subchannel, the original problem is equivalent to independently solving the $N \times S$ subproblems in (12) and (13) for each BS and subchannel. This completes the proof. ■

Accordingly, an optimal user scheduling algorithm under the given power $\boldsymbol{p}$ is easily obtained by

$$I_s^{k,n} = \begin{cases} 1, & \text{if } k = k(n,s) = \arg\max_{k \in \mathcal{K}_n} w_k \cdot r_s^{k,n}(\boldsymbol{p}_s), \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

On the other hand, for any given user scheduling $\boldsymbol{I}$, the original problem reduces to the following power allocation problem:

$$\begin{aligned} \max_{\boldsymbol{p}} \quad & \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}} w_{k(n,s)} \log_2\left(1 + \frac{g_s^{k(n,s),n} p_s^n}{\sum_{m \neq n} g_s^{k(n,s),m} p_s^m + \sigma_s^{k(n,s)}}\right) \\ \text{subject to} \quad & \sum_{s \in \mathcal{S}} p_s^n \leq P^{n,max}, \quad \forall n \in \mathcal{N}, \\ & p_s^n \leq P_s^{n,mask}, \qquad \forall n \in \mathcal{N}, s \in \mathcal{S}. \end{aligned}$$

It appears that any algorithm that globally solves the above problem must have knowledge of all interference channel gains across cells and noise power, forcing it to operate in a centralized fashion. In order to overcome this complexity, we introduce the concept of reference user, where the reference user is the worst user receiving the largest interference from each BS. This idea makes it possible to design fully distributed algorithm requiring only minimal exchange of information between only neighboring BSs and reduce computational complexity significantly. We further consider a practical system restriction that each MS can send its own SINR measurement or corresponding MCS (modulation and coding scheme) level information to the BS once per slot. In other words, multiple feedbacks in a single slot are impossible. Subject to this constraint, we develop the algorithm which avoids multiple loops for power allocation and user scheduling. As we will verify later through extensive analysis and simulations, such a simple approach can obtain efficient enough performance.

The detailed method how each BS n chooses a reference user $k(ref, s)$ on each subchannel s will be presented later. Once the reference user is fixed, each BS tries to find its own power allocation taking into account just one reference user per subchannel instead of solving the above problem considering all (N number of) cochannel users in the network. The problem $(\boldsymbol{P_n})$ to be solved by each BS n can be written as follows. For notational simplify, we suppress the term of scheduled users, $w_s^n = w_{k(n,s)}$, $g_s^n = g_s^{k(n,s),n}$, $g_s^{n,m} = g_s^{k(n,s),m}$, $\gamma_s^n = \gamma_s^{k(n,s),n}$ and $\sigma_s^{k(n,s)} = \sigma_s^n$.

$$\begin{aligned} (\boldsymbol{P_n}) : \max_{\boldsymbol{p}_n} \quad & \sum_{s \in \mathcal{S}} w_s^n \log_2\left(1 + \frac{g_s^n p_s^n}{\sum_{m \neq n} g_s^{n,m} p_s^m + \sigma_s^n}\right) \\ & + \sum_{s \in \mathcal{S}} w_s^{ref} \log_2\left(1 + \frac{g_s^{ref} p_s^{ref}}{\sum_{m \neq ref} g_s^{ref,m} p_s^m + \sigma_s^{ref}}\right) \\ \text{subject to} \quad & \sum_{s \in \mathcal{S}} p_s^n \leq P^{n,max}, \\ & p_s^n \leq P_s^{n,mask}, \qquad \forall s \in \mathcal{S}. \end{aligned}$$

For the given user scheduling and reference user selection, the corresponding power allocation must satisfy the following derived from Karush-Kuhn-Tucker (KKT) conditions:

$$\begin{aligned} p_s^n &= \left[\frac{w_s^n}{\lambda^n \ln 2 + t_s^n} - \frac{\sum_{m \neq n} g_s^{n,m} p_s^m + \sigma_s^n}{g_s^n}\right]_0^{P_s^{n,mask}} \\ \text{where} \quad t_s^n &= \frac{w_s^{ref} g_s^{ref,n} \gamma_s^{ref}(\boldsymbol{p}_s)}{\sum_l g_s^{ref,l} p_s^l + \sigma_s^{ref}}, \ \forall n \in \mathcal{N}, s \in \mathcal{S}. \end{aligned} \tag{17}$$

The operation $[\cdot]_a^b$ is defined by $\min[\max[\cdot, a], b]$. Furthermore, λ_n is a non-negative Lagrange multiplier associated with the total power budget constraint (10) and has to be chosen such that the following complementary slackness condition is satisfied:

$$\lambda_n \Big(\sum_s p_s^n - P^{n,max} \Big) = 0. \tag{18}$$

Clearly, the modified problem $(\boldsymbol{P_n})$ is a still nonconvex. Therefore, (17) and (18) are the first-order *necessary* conditions and there exist a corresponding duality gap between an optimal primal solution. However, encouraged by the asymptotic result [16] that the duality gap becomes zero when the number of subchannels is large, we develop an effective approximation algorithm for the problem $(\boldsymbol{P_n})$ based on these KKT conditions (17)-(18).

Note that a fixed point equation of p_s^n in (17) is a monotonic function of λ^n. Thus, it can be solved via a fast bisection method. Starting from an initial power allocation and λ^n, we calculate the power p_s^n and taxation term t_s^n in (17) for all subchannel. If the sum of updated power exceeds $P^{n,max}$, then λ^n is increased. Otherwise, λ^n is decreased. With the updated power, we repeat this until the equation (18) holds. If no positive value of λ^n matches the equality, then λ^n is set to be zero. In the latter case (interfering too much), BS n does not use all of its available power.

Remark 3.2: Each BS already has knowledge about its direct channel, background noise plus interference through feedbacks from its associating users. The only knowledge each BS n additionally requires is the direct channel, the power, the background noise including interference and the weight of the reference user and the interference channel to the reference user. All of this information about the reference user can be obtained in advance through signaling between neighboring BSs. Note that since the reference user is locally selected, no centralized coordination is necessary and the algorithm can be implemented in a fully distributed fashion.

Remark 3.3: If the term t_s^n is ignored, our power allocation algorithm is reduced to the water-filling (WF) algorithm, where each BS acts selfishly in order to maximize its own performance. By adding this term $t_s^n > 0$, each BS operates in a social way by considering the reference user and lowers the water-filling level, which could lead to a globally better solution.

C. Online Reference User Selection

Based on the notion of reference user, each BS needs to consider the only one user instead of all scheduled users in the network on each subchannel. Denote by $\mathcal{N}(n)$ the neighboring BSs of BS n. From BS n's point of view, although its transmit power will interfere with all the cochannel scheduled users in other BSs, the effect will be marginal for users far from the BS n. On the other hand, the scheduled users from $\mathcal{N}(n)$ will experience large interference from the BS n, and especially the closest user (more precisely, having the largest channel gain to the BS n) will be a dominant victim. Therefore, we propose an online reference user selection method, in which each BS chooses the reference user on each subchannel as follows:

$$k(ref, s) \text{ is the reference user on subchannel } s, \\ \text{where } ref = \arg\max_{m \in \mathcal{N}(n)} g_s^{k(m,s),n}. \tag{19}$$

We emphasize that the reference user is independently selected by each BS for each subchannel, thus they may differ from subchannel to subchannel. The heuristic method has the underlying intuition that protecting the only dominant victim will be efficient enough by protecting indirectly other less dominant users as well. Fig. 1 depicts an example of our online reference user selection procedure.

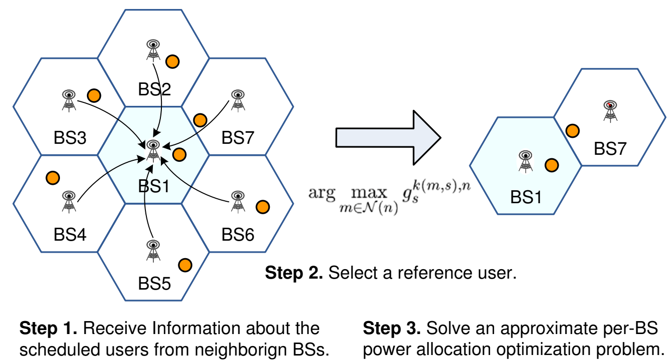


Fig. 1. Online reference user selection method

There may be other methods to select the reference user, e.g., make one virtual user by averaging channels of the scheduled users from neighboring BSs. In this paper, however, we consider only the above heuristic method for the implementation simplicity. Our method can be extended in a straightforward way (e.g., select the M closest users) to include multiple reference users, which only leads to the small increase in complexity. We will discuss the effect of the number of reference users in subsection IV-A.

D. General Algorithm Description

Table I describes a conceptual pseudo-code for the general algorithm of our framework. At each slot, each BS n starts from a proper power allocation. For the given power, the BS executes the user scheduling by (16) and selects the reference users by (19) up to M users. For this given user scheduling and reference user selection, each BS n iteratively updates its own taxation term t_s^n and power $\boldsymbol{p}^n$ until $\boldsymbol{p}$ converges or the maximum number of iterations is reached. And then each BS repeats the user scheduling and the reference user selection for the updated power and goes into power allocation loop again. This procedure is repeated until the user scheduling $\boldsymbol{I}$ converges or the maximum number of iterations is reached. The general algorithm not only has a prohibitively high computational complexity due to inner and outer loops, but also requires multiple information exchanges per slot between BSs to reflect the updated interference level followed by the updated power. To overcome this complexity, we will propose a simplified algorithm in subsection III-G, which

TABLE I
GENERAL ALGORITHM DESCRIPTION

Power initialization
repeat (user scheduling loop):
    User scheduling
    Reference user selection up to M users
    repeat (power allocation loop):
        Taxation update
        Power allocation
    until $\boldsymbol{p}$ converges or max # of iterations is reached
until $\boldsymbol{I}$ converges or max # of iterations is reached

avoids multiple loops with an appropriate initial power and adopts a feedback reduction technique.

E. Initial Power Setting

Our algorithm requires an initial power setting for each slot. Since our problem is a nonconvex problem, different starting points may lead to different solutions with different speeds. The following three strategies for the choice of initial power are carefully investigated in this paper.

- *Uniform power*: This is a static strategy, in which the power allocation always starts from the same point for every slot. Each BS uniformly splits its maximum transmission power to all subchannels, i.e., $p_s^{n,init}(t) = P^{n,max}/S$.
- *Random power*: First, each BS randomly chooses the initial power level for each subchannel between 0 and $P^{n,max}/S$. And then each BS scales it up with an appropriate weight $\frac{P^{n,max}}{\sum_s p_s^{n,init}(t)}$ to use up the total transmission power budget, i.e., $\sum_s p_s^{n,init}(t) = P^{n,max}$.
- *Previous power*: Each BS starts from the power determined in the previous slot, i.e., $p_s^{n,init}(t) = p_s^n(t-1)$.

Through extensive simulations in subsection IV-A, we will demonstrate that using the previous power as an initial power surprisingly works well although channels are time-varying and scheduled users may change at each slot.

F. Feedback Reduction

To determine a reference user, each BS n requires the channel gain $g_s^{ref,n}$ from BS n to the candidates of reference users, i.e., the scheduled users in neighboring cells $\mathcal{N}(n)$. Once the reference user is selected, additional information about the reference user is needed for the execution in the power allocation part. The followings are additionally required parameters of the reference user:

- w_s^{ref}: the weight of the reference user,
- $g_s^{ref} p_s^{ref}$: the received signal strength of the reference user,
- $\sum_{m \neq ref} g_s^{ref,m} p_s^m + \sigma_s^{ref}$: the noise plus interference strength of the reference user.

The above parameters for the candidates users need to be collected by neighboring BSs $\mathcal{N}(n)$ and are forwarded to BS n; however, the per-slot message exchange between BSs may be a burden. Since the parameters for the candidate users except the elected reference user become unnecessary, we now present a more practical solution with reduced feedback.

We reduce the feedback overhead both temporally and spatially. For *temporal feedback reduction*, each BS calculates the average of the above parameters for all candidate users and sends them to its neighboring BSs infrequently, that is, every $T \gg 1$ slots. Each BS should maintain a table containing these averages of candidate users. The only thing that needs to be exchanged at each slot is the indexes of scheduled users. Once each BS receives the indexes of scheduled users from neighboring BSs, then it uses the parameters in the table corresponding to the indexes. For *spatial feedback reduction*, neighboring BSs send the above parameters of only cell edge users. This idea comes from an observation that cell center users are not likely to be selected as the reference user.

TABLE II
SIMPLIFIED PROPOSED ALGORITHM DESCRIPTION

1: Power initialization $\boldsymbol{p}^n$
2: User scheduling according to (16):
$k(n,s) = \arg\max_{k \in \mathcal{K}_n} w_k \cdot r_s^{k,n}(p_s^n)$.
3: Reference user selection according to (19).
4: Taxation update according to (17):
5: Power allocation via bisection:
$[a,b] \leftarrow [0, \lambda_{max}^n]$.
while $|\sum_s p_s^n - P^{n,max}| < \delta$,
Set $\lambda^n = (a+b)/2$ and update $\boldsymbol{p}^n$ according to (17).
if $\sum_s p_s^n > P^{n,max}$, then $[a,b] \leftarrow [\lambda^n, b]$,
else $\sum_s p_s^n < P^{n,max}$, then $[a,b] \leftarrow [a, \lambda^n]$.
end while

G. Simplified Algorithm Description

In this section, we merge components developed in the above subsections, such as user scheduling, power allocation, online reference user selection, initial power setting and feedback reduction, and propose a simplified algorithm.

On the contrary to the general algorithm in Table I, this simplified algorithm in Table II limits the number of reference users to one and avoids multiple loops for power allocation and user scheduling. In other words, our algorithm executes user scheduling and power allocation step-by-step without a loop, which can be done very fast. Also, this requires feedback from each MS just once per slot, which coincide with the practical system constraint. This step-by-step approach is shown to be efficient enough through extensive simulations in section IV when the initial power is set properly.

H. Complexity Analysis

We summarize the computational complexity and inter-BSs signaling complexity of various algorithms in Table III.

Computational complexity consists of the complexity of user scheduling and power allocation. User scheduling has a linear complexity $\mathcal{O}(SK)$ with the number of users for each subchannel for all algorithms except MC-IIWF. For power allocation, EQ has zero complexity. WF can be obtained by both exact algorithm $\mathcal{O}(S)$ and iterative algorithm with error margin ϵ is $\mathcal{O}\left(\log_2 \frac{P_{max}}{\epsilon}\right)$ [21]. The only difference between WF and our proposed algorithm is the taxation term considering the reference user. Thus, the complexity of power allocation for our proposed algorithm is basically the same as that for WF. MGR (Multi-sector GRadient) in [18], one of the state-of-the-art dynamic interference management algorithms, adjusts the power allocation slowly (every n_p) and condenses the complexity for updating power to $1/n_p$. However, MGR brings about an additional complexity from a virtual scheduling that needs to be run n_v times per slot. Accordingly, the total computational complexity for power allocation is high, $\frac{1}{n_p}\mathcal{O}(S) + n_v \mathcal{O}(SK)$. Another recently developed MC-IIWF (MultiCell Improved Iterative Water Filling) in [17] is the centralized algorithm that has iteration loops for power allocation and user scheduling. Let T_1 be the number of iterations needed for iteration loops. Then the total computational complexity is equal to $T_1 \cdot (\mathcal{O}(SK) + \mathcal{O}(SN))$.

Now let us investigate the inter-BSs signaling complexity. EQ and WF do not require any inter-BS signaling over-

TABLE III
COMPLEXITY COMPARISON OF VARIOUS ALGORITHMS

Algorithm	Computational complexity		Signaling complexity (inter-BSs)	
	User scheduling	Power allocation	Per-slot feedback	Periodic feedback
EQ	$\mathcal{O}(SK)$	Zero	*Zero*	*Zero*
WF	$\mathcal{O}(SK)$	$\mathcal{O}(S)$ or $\mathcal{O}\left(\log_2 \frac{P_{max}}{\epsilon}\right)$	*Zero*	*Zero*
MGR	$\mathcal{O}(SK)$	$\frac{1}{n_p}\mathcal{O}(S) + n_v\mathcal{O}(SK)$	*Zero*	$\|N_n\|S$
Proposed	$\mathcal{O}(SK)$	$\mathcal{O}(S)$ or $\mathcal{O}\left(\log_2 \frac{P_{max}}{\epsilon}\right)$	ρS	$\rho\|\mathcal{K}_n\|AS$
MC-IIWF	$T_1 \cdot (\mathcal{O}(SK) + \mathcal{O}(SN))$		Complete information is assumed.	

head because they are autonomous algorithms not considering neighboring BSs at all. MGR adjusts the power allocation slowly so that it requires not per-slot but periodic feedback, $|N_n|S$ (sensitivity information for neighboring BSs and all subchannels). MC-IIWF is a centralized algorithm and assumes the central control unit to have complete information. The proposed algorithm requires the periodic feedback about the candidate users for reference users, $\rho|\mathcal{K}_n|AS$, where ρ is the average portion of edge users and $A = 3$ is the number of additionally required information about the reference user. This can be easily done through high-speed dedicated lines between BSs. A per-slot feedback of the indexes of scheduled users for each subchannel is required as well.

In brief, the computational complexity of the proposed algorithm is the same as that of WF and is much lower than that of state-of-the-art dynamic interference management algorithms such as MGR and MC-IIWF. For signaling complexity, although the feedback per slot-wise manner is challenging, the parameters need to be exchanged between neighboring BSs at each slot are just the indexes of scheduled users. we believe that such a simple information can be exchanged through high-speed dedicated lines or by introducing mini-slot for message exchange at the head of the slot.

IV. Performance Evaluation

A two-tier multi-cell network composed of hexagonal 19 cells is basically considered for simulations, where the distance between BSs is 2km and a wrap-around technique is adopted for creating the same number of interfering cells around every one of the 19 cells. In order to provide more realistic simulation results, we also consider a real 3G BS deployment topology in the subsection IV-C and a heterogeneous network topology in the subsection IV-D. The system load is 20 users per cell and they are uniformly distributed in each cell. All users are assumed to have a logarithmic utility function, i.e., $U(R_k) = \log R_k$, but other utility functions enforcing more fairness are also considered in our technical paper in [22].

We consider a system having 6 subchannels each of which consists of multiple subcarriers. The maximum transmit powers of BSs are all the same with 43dBm. Channel models follow ITU PED-B path loss model and Jakes' Rayleigh fading model. The channel bandwidth and the time-slot length are set to be 10MHz and 1ms, respectively. The other parameters for simulations follow the suggestions in the IEEE 802.16m evaluation methodology document [23]. All simulations are run over 5000$\sim$10000 slots.

Our proposed algorithm is compared to conventional equal power allocation (EQ) and selfish water-filling (WF), as well as recently developed MGR [18] and MC-IIWF [17]. As performance metrics, the geometric average of user throughputs (GAT) and the average of edge user throughputs (AET) are used. We use GAT since maximizing this metric is equivalent to our system objective (sum of log throughputs). We treat the AET as the 5th percentile average throughput is equal to the average of the lowest 5% throughput of users.

A. Verification of Proposed Algorithm

We first verify the performance of the proposed algorithm by varying several tunable parameters.

Fig. 2(a) shows the GAT performance of the proposed algorithm for the number of reference users per subchannel and that of EQ as a baseline. As mentioned in Remark 3.3, the case without a reference user can be regarded as WF. Thus, our proposed algorithm taking reference users into consideration can obtain higher performance gain. It is noteworthy that considering only the one reference user per subchannel is efficient enough because it can obtain more than 97% of the performance considering all the six neighbors.

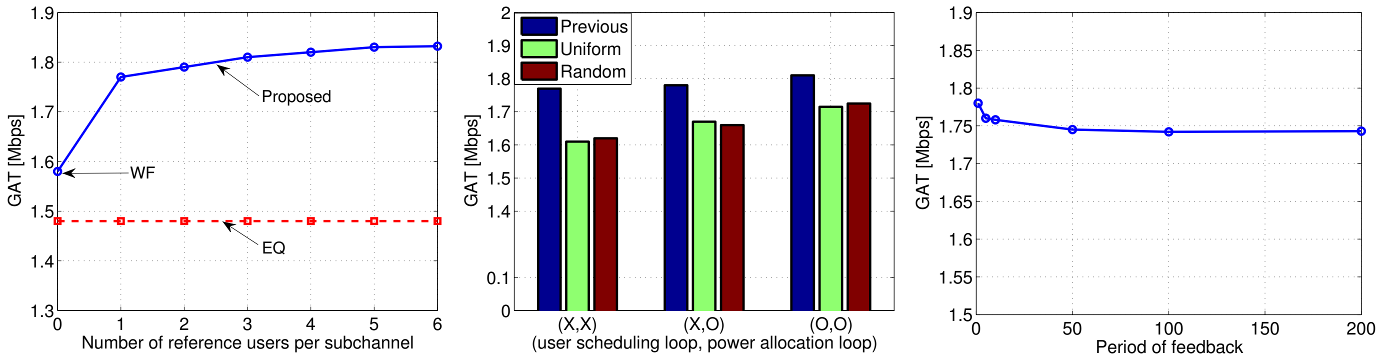
Fig. 2(b) shows the effect of iteration loops and initial power: (i) adding user scheduling loop and/or power allocation loop give an additional performance gain from any initial power setting and (ii) using power at the previous slot as an initial power performs better than other two strategies. However, for the case in which power at the previous slot is used as an initial power, the performance gain from adding power allocation loop is marginal. It is because in some sense using the previous power has the effect of iterations not in a slot but in a slot-by-slot manner. Fig. 2(b) is an encouraging result that leads us to design the algorithm without loops and use the previous power as an initial power.

We also test the effect of infrequency feedback instead of slot-by-slot feedback about reference users in Fig. 2(c). Even though we reduce feedback amount significantly by choosing a long feedback period such as 100$\sim$200 slots, performance loss is relatively small.[3] This implies that considering reference users based on the averaged information is good enough.

B. Performance Comparison with Other Algorithms

We now compare the performance of proposed algorithm with other four algorithms: conventional EQ, selfish WF, MGR

[3] If we take the mobility of user into consideration, the GAT performance will eventually decrease due to the error of channel estimation as the period of feedback increases. However, we believe that the error associated with about 100 slots will be very marginal because the user can move just 0.1$\sim$2m (depending on the speed of the user, e.g., 3$\sim$72km/h).



(a) The number of reference users

(b) Iteration loops and initial power

(c) Feedback period

Fig. 2. Effect of several tunable parameters

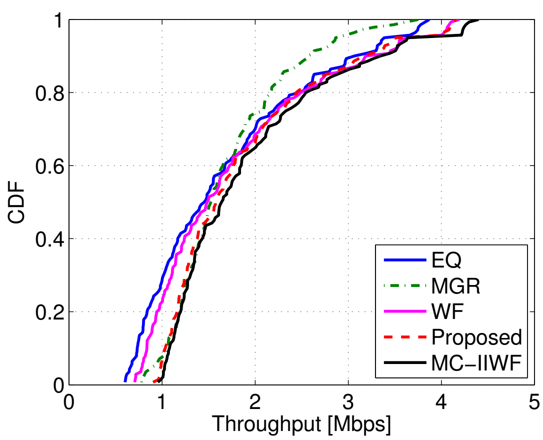


Fig. 3. Comparison with other algorithms.

[18] which adjusts power allocation slowly than per-slot basis user scheduling, and MC-IIWF [17] which is a centralized algorithm achieving the near-optimal performance.

We plot the CDF (cumulative distribution function) of user throughputs in Fig. 3. Compared to EQ, WF and MGR, our proposed algorithm can improve the throughputs for all users in the network. Particularly, we can observe a higher improvement (more than 50% improvement of AET) for users achieving low throughputs, i.e., located at cell edges. This is due to the fact that ICI management is mainly targeted for performance improvement of edge users. In addition, our proposed algorithm can achieve about 95% of the performance of near-optimal MC-IIWF in terms of two representative throughput metrics (GAT and AET) as well as the arithmetic average of user throughputs. It is very surprising that such a simple distributed algorithm can obtain a similar performance to the centralized algorithm that is hard to implement due to prohibitive complexity.

C. Topology with Real BS deployment

In order to provide more realistic results, we test the performance of our algorithm under a real BS deployment topology. The map of BS layout that we use for the performance

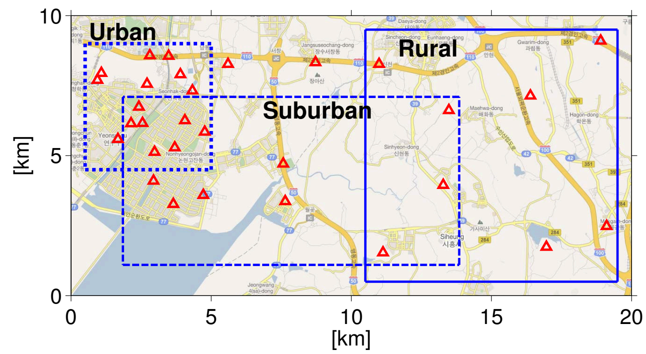


Fig. 4. Real 3G BS deployment map.

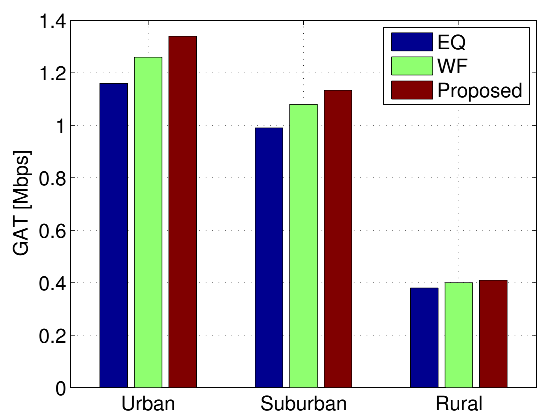
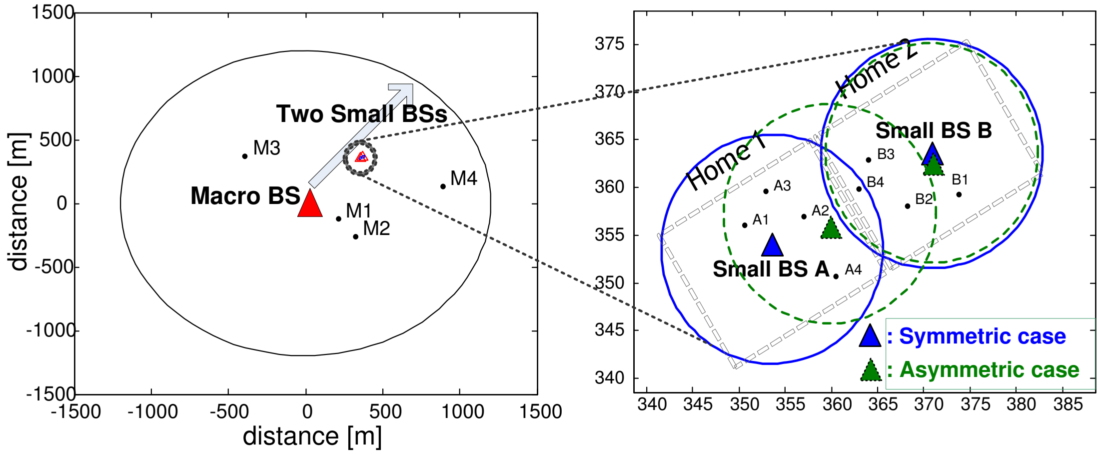
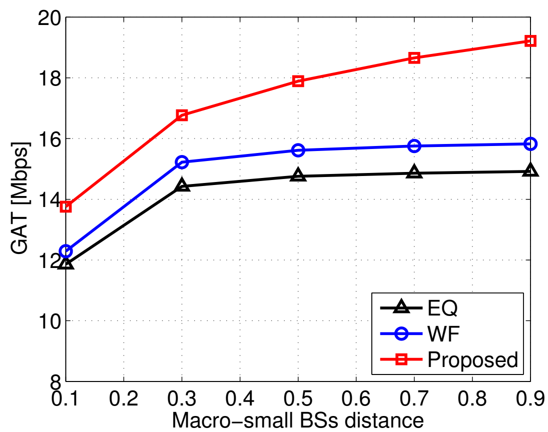


Fig. 5. GAT performance in real BS deployment topology.

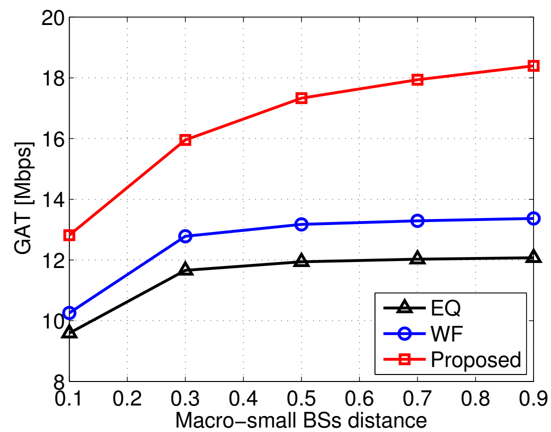
evaluation of our algorithms is presented in Fig. 4. It is a part of 3G network operated by one of the major service providers in Korea. There are a total of 30 base stations within a 20×10 km^2 rectangular area. We assume that the number of BSs per unit area is proportional to the user density. In other words, we assume that the average number of users per cell is almost similar because BSs in an urban environment cover a small area and BSs in a rural environment a large area. Under this assumption, we generate users one-by-one in the rectangular area and attach them to the closest BS until



(a) Heterogeneous network topology (one macro BS + two small BSs)



(b) Symmetric case



(c) Asymmetric case

Fig. 6. Performance in the heterogeneous network topology.

all the BSs in the network have 20 users.

We district three different zones: urban (15 BSs in 4.5×4.5 km^2), suburban (15 BSs in 12×6 km^2) and rural (8 BSs in 9×9 km^2) areas[4]. Fig. 5 shows the GAT performance under urban, suburban and rural environment. As expected, we can obtain a high performance gain in the urban and suburban environments. On the other hand, however, almost low or no gain is found in the rural environment, which means that the ICI management is not essential in a sparse topology.

Another nice feature of our proposed algorithm is a possibility for *incremental deployment*. Suppose that we implement our algorithm only on the BSs in a specific area. While the BSs inside this area performs well as we want, the BSs in the boundary of the area does not. This is because they may not receive information about reference users from the some of its neighboring BSs on which our algorithm is not implemented. Even such a case, our algorithm will automatically reduce to WF. Thus, it performs like WF at least and better than EQ. Compared to the full deployment case, the partial deployment case where only 15 BSs (mainly selected from the urban areas among 30 BSs) are equipped with the proposed algorithm can achieve more than 85% gain. The result encourages the service provider to upgrade its BSs incrementally from the urgent ones, e.g., densely located BSs interfering severely each other.

In the scenarios that have been shown so far, the proposed algorithm could obtain about 15∼20% performance gain in terms of GAT (but more than 50% in terms of AET) compared to EQ. If we evaluate the performance in a heterogeneous network topology or adopt other utility functions enforcing more fairness, the gain becomes more substantial. Due to limited space, we only provide the interesting scenario of the heterogeneous network in the next subsection. Please refer to our technical paper [22] for further details.

[4]In order to see clearly how the density of BSs affects the performance gain, the distance between BSs in suburban and rural zones are increased by 1.5 and 2 times, respectively.

D. Heterogeneous Network (Macro + Small Base Stations)

Consider a heterogeneous network topology having two small BSs inside a macro cell as shown in Fig. 6(a). These small BSs provide a high-speed indoor access mainly to home or office users. In order to reflect the ICI not only between macro-small cells but also between small cells, we assume that two small cells are adjacent with each other. Two deployment cases of small BSs are considered: (i) symmetric case - each small BS is located in the center of the home and (ii) asymmetric - the small BS A in home 1 is located at the border between homes. We randomly generated four users in each small cell and macro cell as well. The maximum transmit power and the radius of small BSs are set to be 15dBm and 20m, respectively. For modeling the propagation environment in small cells, an indoor propagation loss model $37+32\log_{10}(d[m])$ is adopted. We ran simulations by varying the distance between macro and small BSs.

Fig. 6(b) shows GAT performance achieved by EQ, WF and the proposed algorithm in the symmetric case. When small cells are close to the macro BS, there exists both inter-macro-small-cell interference and the inter-small-cell interference. If the small cells move far away from the macro BS, then the inter-macro-small-cell interference decreases drastically so that the overall performance grows as the distance between macro and small BSs increases. In the case of long distance between macro and small BSs, the performance is mainly bottlenecked by the inter-small-cell interference so that mitigating this dominant interference will give the high performance gain. Compared to EQ, The proposed algorithm can achieve 15∼34% performance gain depending on the distance between macro and small BSs.

Fig. 6(c) shows results in the asymmetric case. As a whole, overall trends are similar to those the symmetric case in Fig. 6(b), but the performance gain achieved by the proposed algorithm can be increased up to 52%. This is because that the ICI between small cells becomes more severe in the asymmetric case than in the symmetric case.

V. Conclusion

In order to overcome low performance of conventional static interference management algorithms and high complexity of existing dynamic interference management algorithms, this paper focused on developing a distributed power control and multi-user scheduling algorithm that has low complexity, yet achieves high efficiency. The main idea of this paper is to consider only one reference user, which is shown to be efficient enough (i.e., a good approximation) and makes it possible to design a fully distributed algorithm with minimal exchange of information between only neighboring BSs. We reduced the computational complexity by avoiding multiple loops for power allocation and user scheduling with an appropriate initial power setting. We also further reduced the signaling complexity both temporally and spatially. Through extensive simulations and complexity analysis, we showed that the proposed algorithm not only performs well but also is practically implementable.

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