

Rumor Source Detection under Querying with Untruthful Answers

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Abstract—Social networks are the major routes for most individuals to exchange their opinions about new products, social trends and political issues via their interactions. It is often of significant importance to figure out who initially diffuses the information, *i.e.*, finding a rumor source or a trend setter. It is known that such a task is highly challenging and the source detection probability cannot be beyond 31% for regular trees, if we just estimate the source from a given diffusion snapshot. In practice, finding the source often entails the process of querying that asks “Are you the rumor source?” or “Who tells you the rumor?” that would increase the chance of detecting the source. In this paper, we consider two kinds of querying: (a) simple *batch* querying and (b) *interactive* querying with *direction* under the assumption that querees can be untruthful with some probability. We propose estimation algorithms for those queries, and quantify their detection performance and the amount of extra budget due to untruthfulness, analytically showing that querying significantly improves the detection performance. We perform extensive simulations to validate our theoretical findings over synthetic and real-world social network topologies.

I. INTRODUCTION

Information spread is universal in many types of on-line/offline and social/physical networks. Examples include the propagation of infectious diseases, the technology diffusion, the computer virus/spam infection in the Internet, and tweeting and retweeting of popular topics. Finding a “culprit” of the information spreading is of great significance, because, for a harmful diffusion, its spreading can be mitigated or even blocked by vaccinating humans or installing security updates. Detecting the rumor source has been regarded as a challenging task unless sufficient side information is provided. The seminal work by Shah and Zaman [1] analytically provides the detection performance of the MLE (maximum-likelihood estimator) under regular tree topologies, where the detection probability is upper-bounded by 31% if the number of infected nodes goes to infinity and much less for other practical topologies. Since then, extensive attentions have recently been made in various types of network topologies and diffusion models [2]–[5], whose major interests lie in constructing an efficient estimator and providing theoretical limits on the detection performance.

In practice, the effort of finding the rumor source is made in conjunction with extra processes with the aim of obtain-

ing more side information and thus improving the detection performance. In this paper, we aim at quantifying the impact of querying, where querying refers to the process of asking some questions. Obviously, it is expected that such queries improves the detection performance, but little attention has been made to quantification of the detection performance in presence of querying. In literature, it has been studied what happens if multiple snapshot observations are provided [6], or if a restricted node subset (also called a suspect set) is given [7] a priori.

In this paper, we study the impact of querying in a highly generalized setup. Users¹ may be untruthful with some probability, where two different types of querying are considered: (a) *simple batch querying* and (b) *interactive querying with direction*. In simple batch querying, for a given querying budget, a candidate queree set is first chosen, and the question of “Are you the rumor source?” (referred to as *identity question*) can be asked to the querees in the set multiple times. Due to limited budget, a source estimation algorithm should strike a good balance between the size of the candidate set and the number of questions, depending on the amount of budget, the degree of untruthfulness, and the underlying graph topology. In interactive querying with direction, we start with some initial queree, and iteratively ask a series of questions “Are you the rumor source?” and “If not, who spreads the rumor to you?” (referred to as *direction question*) to the current queree, and determine the next queree, using the (possibly untruthful) answers for the second question, where this iterative querying process lasts until the entire querying budget K is exhausted. A source estimation algorithm in this query type should smartly consider the tradeoff between the number of questions and the number of querees we can ask.

We propose the estimation algorithms for both types of queries and analyze their detection performances as well as the minimum budget to satisfy an arbitrary detection performance. We summarize our main contributions as follows:

- *Simple batch querying*. We first formulate an optimization problem that maximizes the detection probability over the number of questions to be asked, the candidate queree set, and the estimators for a given diffusion snapshot and the answer samples. We discuss its analytical challenges and propose an approximate estimation algorithm that selects the candidate queree set based on the hop-distance from the rumor center and the MLE for the “filtered” candidate nodes by including only nodes

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¹In this paper, the terms “user” and “queree” are used interchangeably.

with many positive answers. We prove that for a given probability $p > 1/2$ that users are truthful, d -regular tree, and a budget K , for any $0 < \delta < 1$, if $K \geq \frac{c(2/\delta)}{p^2 \log(\log(2/\delta))}$ with some constant c (depending on the degree d) then the detection probability is at least $1 - \delta$.

- **Interactive querying with directions.** In this querying type, we also consider an optimization problem that maximizes the detection probability and discuss the technical challenges in solving it, where we assume that users are only untruthful for the direction question for simplicity. We propose an estimation algorithm that starts with a “rumor center” as the initial querier, and apply a kind of majority rule in determining the next querier, *i.e.*, selecting the node with highest vote for multiple direction questions. We analyze this simple, yet powerful estimation algorithm and characterize its detection probability for given parameters. From this, we establish the minimum budget for any given detection probability: for the probability $q > 1/d$ that users are truthful for a direction question, d -regular tree, and a given budget K , for any $0 < \delta < 1$, if $K \geq \frac{c \log(7/\delta)}{q^3 \log(\log(7/\delta))}$ with some constant c (which depends on the degree d), then the detection probability is at least $1 - \delta$. This result quantifies the power of the direction question in addition to the identity question, reducing the required budget to satisfy the detection probability $1 - \delta$ by a logarithmic factor with respect to the scaling of $1/\delta$ for small $\delta > 0$.
- **Evaluation over synthetic and real-world graphs.** Our analytical results above provide useful guidelines on how much budget is required to guarantee a given detection performance for different querying types when users are untruthful. We validate our findings via extensive simulations over popular random graphs (*Erdős-Rényi* and scale-free graphs) and a real-world Facebook network. As an example, in Facebook network, the interactive querying requires about 200 queries to achieve almost one detection probability when $q > 0.5$ because the tracking by the direction is efficient due to the small diameter of the network.

Related work. The research on rumor source detection has recently received significant attentions. The first theoretical approach was done by Shah and Zaman [1], [2], [8] and they introduced the metric called *rumor centrality*, which is a simple topology-dependent metric. They proved that the rumor centrality describes the likelihood function when the underlying network is a regular tree and the diffusion follows the SI (Susceptible-Infected) model, which is extended to a random graph network in [9]. Zhu and Ying [3] solved the rumor source detection problem under the SIR (Susceptible-Infected-Removed) model and took a sample path approach to solve the problem, where a notion of *Jordan center* was introduced, being extended to the case of sparse observations [10]. The authors of [5], [11] and [12] studied the problem of estimating the source for random growing trees, where unlike aforementioned papers, they did not assume an un-

derlying network structure. The authors in [13] inferred the historical diffusion traces and identifies the diffusion source from partially observed cascades, and similarly in [14], partial diffusion information is utilized. Recently, there has been some approaches for the general graphs in [15], [16] to find the information source of epidemic. All the detection mechanisms so far correspond to point estimators, whose detection performance tends to be low. There was several attempts to boost up the detection probability. Wang *et al.* [6] showed that observing multiple different epidemic instances can significantly increase the the detection probability. Dong *et al.* [7] assumed that there exist a restricted set of source candidates, where they showed the increased detection probability based on the MAPE (maximum a posterior estimator). Choi *et al.* [17], [18] showed that the anti-rumor spreading under some distance distribution of rumor and anti-rumor sources helps finding the rumor source by using the MAPE. The authors in [5], [19], [20] introduced the notion of *set estimation* and provide the analytical results on the detection performance. These are close to our work, where querying is considered in detecting the rumor source. However, our work is done in a much more generalized and practical setup in the sense that we consider the case when users may be untruthful, and also two types of practical querying scenarios are studied.

II. MODEL AND PRELIMINARIES

A. Model

Rumor diffusion. We describe a rumor spreading model which is commonly adopted in other related work, *e.g.*, [1]–[3]. We consider an undirected graph $G = (V, E)$, where V is a countably infinite set of nodes and E is the set of edges of the form (i, j) for $i, j \in V$. Each node represents an individual in human social networks or a computer host in the Internet, and each edge corresponds to a social relationship between two individuals or a physical connection between two Internet hosts. We assume a countably infinite set of nodes for avoiding the boundary effect. As a rumor spreading model, we consider a SI model, where each node is in either of two states: *susceptible* or *infected*. All nodes are initialized to be susceptible except the rumor source, and once a node i has a rumor, it is able to spread the rumor to another node j if and only if there is an edge between them. Let a random variable τ_{ij} be the time it takes for node j to receive the rumor from node i if i has the rumor. We assume the τ_{ij} are exponentially distributed with rate $\lambda > 0$ independently of everything else. Without loss of generality, we assume that $\lambda = 1$. We denote $v_1 \in V$ by the rumor source, which acts as a node that initiates diffusion and denote $V_N \subset V$ by N infected nodes under the observed snapshot $G_N \subset G$. In this paper, we consider the case when G is a regular tree and our interest is when N is large, as done in many prior work [1], [2], [6], [7], [19].

Querying with untruthful answers. A detector is allowed to query the nodes, where querying refers to a process of asking some questions (which will be shortly clarified depending on querying scenarios). The detector is given some querying budget K , where we assume that one budget is used to ask

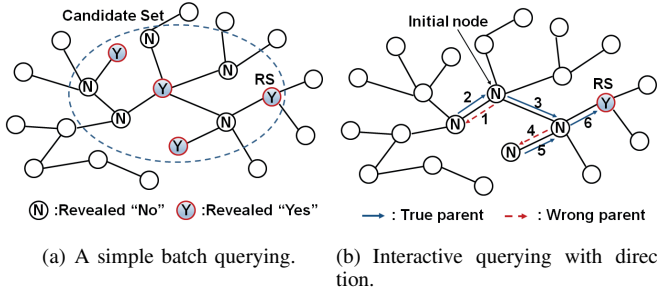


Fig. 1. Examples of two querying types with untruthful answers. In (a), the querier selects a candidate set (a dotted circle) and asks just one identity question (*i.e.*, $r = 1$) in a batch. In (b), starting from the initial node, the querier first asks “Are you the rumor source?”, and if no, she further asks “Who spreads the rumor to you?”, where the querier interactively tracks the true source, but the queryees may be untruthful (RS: Rumor Source).

one question. In this paper, we consider two types of queries (see Fig. 1), where a queryee may be untruthful, as modeled in what follows:

- **Simple batch querying.** This query is parameterized by r , where a querier first chooses K/r (K is a multiple of r for expositional convenience) candidate nodes to each of which an *identity question* of “Are you the rumor source?” is asked r times. We call r the *repetition count* throughout this paper. Each queryee v is truthful in answering each question only with probability p_v , *i.e.*, even if she is the rumor source, she lies with probability $1 - p_v$. We assume that the answers are independent over the r queries across the queryees. We assume the homogeneous case when $p_v = p$ for all $v \in V_N$, and $p > 1/2$,² *i.e.*, all users are biased with truthful answers. For example, Fig 1(a) shows a candidate set of nodes inside a dotted circle, where with $r = 1$ we have four “yes” nodes and five “no” nodes.
- **Interactive querying with direction.** In this querying type, there are two questions, where one is the identity question, as in the simple batch querying and another is the *direction question* of “Who spreads the rumor to you?.” This query is also parameterized by r , where the querying process occurs in the following interactive manner: A querier first chooses an initial node to ask the identity question, and further asks the direction question r times (*i.e.*, repetition count), if the queryee answers that she is not the rumor source. The querying process stops when the queryee says that she is the rumor source or the entire budget K is exhausted. A querier determines the next queryee from r direction questions, where each queryee v is truthful for the direction question only with probability q_v .³ In other words, she lies for the direction question with probability $1 - q_v$ and designates a node uniformly at random out of all neighbors except for the node who truly spreads the rumor to v as a bogus “parent”. Then, the querier chooses one of her neighbors as the parent

²In the case $p < 1/2$, one can flip the sign of the final estimation to achieve the same guarantee.

³We model untruthfulness only in the direction question, because we aim at purely focusing on the impact of untruthfulness in the interactive search of the true rumor source, which seems to be the critical component in this type of querying.

of v and repeats the same procedure. As in the simple batch querying, we assume the homogeneity in truthfulness that $q = q_v$ for all $v \in V_N$, and $q > 1/d$, *i.e.*, users’ bias for truthful answers⁴. Fig 1(b) shows an example scenario that starting from the initial node, a sequence of nodes answer the interactive queries truthfully or untruthfully.

Goal. Our goal is to propose efficient estimation algorithms that are practically implementable, for both types of queries with users’ untruthfulness. Especially, we aim at theoretically quantifying the detection performance of our proposed algorithms by providing the lower bound of the required budget K to satisfy any arbitrary detection probability, *i.e.*, the sufficient budget K for the target detection quality.

B. Preliminaries: Rumor Centrality

As a preliminary, we explain the notion of *rumor centrality*, which is a graph-theoretic score metric and is originally used in detecting the rumor source in absence of querying and users’ untruthfulness, see [1]. This notion is also importantly used in our framework as a sub-component of the algorithms for both simple batch querying and interactive querying with direction. In regular tree graphs, Shah and Zaman [1] showed that the source chosen by the MLE becomes the node with highest rumor centrality. Formally, the estimator chooses v_{RC} as the rumor source defined as

$$\begin{aligned} v_{RC} &= \arg \max_{v \in V_N} \mathbb{P}(G_N | v = v_1) \\ &= \arg \max_{v \in V_N} R(v, G_N), \end{aligned} \quad (1)$$

where v_{RC} is called *rumor center* and $R(v, G_N)$ is the rumor centrality of a node v in V_N . The rumor centrality of a particular node is calculated only by understanding the graphical structure of the rumor spreading snapshot, *i.e.*, $R(v, G_N) = N! \prod_{u \in V_N} (1/T_u^v)$ where T_u^v denotes the number of nodes in the subtree rooted at node u , assuming v is the root of tree G_N (see [1] for details).

III. DETECTION USING SIMPLE BATCH QUERIES

A. Algorithm based on MLE and Rumor Centrality

A source estimation algorithm for simple batch queries consists of the following steps: We first need to appropriately choose the repetition count r and the candidate set $C_r \subset V_N$ of size K/r and ask the queries to the nodes in C_r . Then, we will be given a sample of the answers, which we denote by a vector $A_r := A_r(p) = [x_1, x_2, \dots, x_{K/r}]$ with $0 \leq x_i \leq r$ representing the number of “yes” answers of the i -th node of C_r . Then, it is natural to consider an algorithm based on MLE, to maximize the detection probability, that solves the following optimization:

$$\text{OPT-S: } \max_{1 \leq r \leq K} \max_{C_r} \max_{v \in C_r} \mathbb{P}[G_N, A_r | v = v_1], \quad (2)$$

where the inner-most max corresponds to the MLE given the diffusion snapshot G_N and the query answer sample A_r .

⁴In d -regular trees, when $q = 1/d$, the answer for the direction question turns out to be uniformly random.

Challenges. We now explain the technical challenges in solving **OPT-S**. To that end, let us consider the following sub-optimization in **OPT-S** for a fixed $1 \leq r \leq K$:

$$\text{SUB-OPT-S: } \max_{C_r} \max_{v \in C_r} \mathbb{P}[G_N, A_r | v = v_1]. \quad (3)$$

Then, the following proposition provides the solution of **SUB-OPT** whose proof is provided in our technical report [21].

Proposition 1: Construct C_r^* by including the K/r nodes in the decreasing order of their rumor centralities. Then, C_r^* is the solution of **SUB-OPT-S**.

Despite our knowledge of the solution of **SUB-OPT-S**, solving **OPT-S** requires an analytical form of the objective value of **SUB-OPT-S** for C_r^* to find the optimal repetition count, say r^* . However, analytically computing the detection probability for a given general snapshot is highly challenging due to the following reasons. We first note that

$$\begin{aligned} & \max_{v \in C_r} \mathbb{P}[G_N, A_r | v = v_1] \\ &= \underbrace{\mathbb{P}[v_1 \in C_r^*]}_{(a)} \times \underbrace{\max_{v \in C_r^*} \mathbb{P}[G_N, A_r | v = v_1, v_1 \in C_r^*]}_{(b)}. \end{aligned} \quad (4)$$

First, the term (a) is difficult to analyze, because only the rumor center allows graphical and thus analytical characterization as discussed in Section II-B, but other nodes with high rumor centrality is difficult to handle due to the randomness of the diffusion snapshot. Second, in (b), we observe that using the independence between G_N and A_r , by letting the event $\mathcal{A}(v) = \{v = v_1, v_1 \in C_r^*\}$,

$$\begin{aligned} \hat{v} &= \arg \max_{v \in C_r^*} \mathbb{P}[G_N, A_r | \mathcal{A}(v)] \\ &= \arg \max_{v \in C_r^*} \mathbb{P}[A_r | \mathcal{A}(v)] \times \mathbb{P}[G_N | \mathcal{A}(v)]. \end{aligned} \quad (5)$$

Then, the node \hat{v} maximizing (b) is the node v that has the maximum *weighted* rumor centrality where the weight is $\mathbb{P}[A_r | \mathcal{A}(v)]$. As opposed to the case of characterizing the rumor center in the non-weighted setup [1], analytically obtaining or graphically characterizing \hat{v} in this weighted setup is also hard due to the randomness of the answer for querying, thus resulting in the challenge of computing r that maximizes the detection probability in **OPT-S**.

One can numerically solve **OPT-S**, which, however, needs to generate a lot of A_r samples (one sample requires a vector of answers for K questions). Motivated by this, we propose an algorithm producing an approximate solution of **OPT-S**. The key of our approximate algorithm is to choose C_r that allows us to analytically compute the detection probability for a given r so as to compute a good r easily, yet its performance is close to that of **OPT-S**, as numerically validated in Section VI.

B. Algorithm based on Hop Distance and Majority Rule

We now propose an algorithm that overcomes the aforementioned challenges in (a) and (b) of (4). The key idea is that for (a) we adopt a *hop-distance based selection* of the

Algorithm 1: SB-Q(r), r : Repetition Count

Input: Diffusion snapshot G_N , budget K , degree d , and truthfulness probability $p > 1/2$

Output: Estimator \hat{v}

- 1 $C_r = \hat{V} = \emptyset$;
- 2 Calculate the rumor centrality $R(v, G_N)$ for all $v \in G_N$ as in [1] and let $s \leftarrow \arg \max_{v \in V_N} R(v, G_N)$;
- 3 Construct a candidate set C_r by including each node v that satisfies $d(v, s) \leq l$, where $l = \frac{\log(\frac{K(d-2)}{rd} + 2)}{\log(d-1)}$ and $d(v, s)$ is the hop distance between nodes v and s ;
- 4 **for** each $v \in C_r$ **do**
- 5 Count the number of “yes” (*i.e.*, I am the rumor source) for the identity question, stored at $\mu(v)$, and if $\mu(v)/r \geq 1/2$ then include v in \hat{V} ;
- 6 **while** $K - r|C_r| \geq r$ **do**
- 7 Select a node v satisfying $d(v, s) = l + 1$ uniformly at random;
- 8 Do the same procedure in Lines 5;
- 9 **if** $\hat{V} = \emptyset$ **then**
- 10 $\hat{v} \leftarrow \arg \max_{v \in C_r} R(v, G_N)$;
- 11 **else**
- 12 $\hat{v} \leftarrow \arg \max_{v \in \hat{V}} R(v, G_N)$;

candidate set C_r and for (b) we simply apply a *majority-based rule*.

We first formally describe our algorithm in **SB-Q**(r) parameterized by a repetition counter r , and explain how it operates, followed by presenting the rationale behind it: we first calculate the rumor centrality of all nodes in G_N (Line 2), where s is set to be the rumor center. Then, using the parameter r , we construct the candidate set C_r by the nodes within the hop-distance l given in Line 3 from the rumor center s using the relation that $K \geq r|C_r| = r \frac{d(d-1)^l - 2}{d-2}$ (Line 3). Next, for each node v in C_r , we ask the identity question r times, and count the number of “yes”es in $\mu(v)$ (Lines 4-5). Using this, we filter out the candidate set C_r and construct \hat{V} by including the nodes with $\mu(v)/r \geq 1/2$ (majority rule), *i.e.*, the nodes with higher chance to be the rumor source. When K is not a multiple of r , we handle the remaining $K - r|C_r|$ nodes as in Lines 6-8. Finally we choose the node in \hat{V} with highest rumor centrality, where if $\hat{V} = \emptyset$, we simply do the same task for C_r .

Rationale. We now provide the rationale of **SB-Q**(r) from the perspective of how we handle the analytical challenges in (4) so as to solve **OPT-S** in an approximate manner.

- *Hop Distance based C_r selection:* Selecting C_r based on the distance from the rumor center, rather than based on the sorted rumor centrality permits us to have the closed form of $\mathbb{P}[v_1 \in C_r]$. However, it is not difficult to obtain the lower bound of this probability when we consider the hop distance based C_r by using some preliminary results in [2], [19]. Furthermore, the authors in [19] shows the

probability of distance between rumor source and center decays exponential with respect to the distance, *i.e.*, the source is nearby the rumor center with high probability. Hence, it is a good approximation to the centrality based C_r with analytical guarantees.

- *Construction of the filtered set \hat{V} from querying:* Consider the answer sample of node v for r questions, x_v ($1 \leq x_v \leq r$), where one can easily check that for $x_v \geq r/2$ then the weight $\mathbb{P}[A_r|\mathcal{A}(v)]$ becomes larger than that for $x_v < r/2$ due to $p > 1/2$. We use an approximated version of the weight from the answer samples by setting $\mathbb{P}[A_r|\mathcal{A}(v)] = 1$ if $x_v \geq r/2$, and $\mathbb{P}[A_r|\mathcal{A}(v)] = 0$ if $x_v < r/2$.

Using the above techniques, we are able to have an approximate, but closed form solution of **SUB-OPT-S**, which allows us to compute the best r^* that maximizes the detection probability under such an approximation, as r^* is given in the next section.

C. Detection Performance

We now provide analytical results on the detection performance of **SB-Q**(r). We first start by presenting the lower bound of the detection probability for a given repetition count r in Proposition 2.

Proposition 2: For d -regular trees ($d \geq 3$), a snapshot G_N , a given budget K , our estimator $\hat{v}(G_N, r)$ from **SB-Q**(r) has the following lower-bound of the detection probability:

$$\lim_{N \rightarrow \infty} \mathbb{P}[\hat{v}(G_N, r) = v_1] \geq \left(\frac{r+p}{r+1} \right) \cdot \left(1 - c \cdot \exp\left(\frac{-h_d(K, r)}{2} \right) \right), \quad (6)$$

where $c = 7(d+1)/d$ and

$$h_d(K, r) := \frac{\log\left(\frac{K}{r}\right)}{\log(d-1)} \log\left(\frac{\log\left(\frac{K}{r}\right)}{\log(d-1)}\right).$$

The proof is presented in Section V. The second term of RHS of (6) is the probability that the source is in the candidate set for given K and r . Hence, one can see that for a fixed K , large r leads to the decreasing detection probability due to the smaller candidate set. However, increasing r positively affects the first term of RHS of (6), so that there is a trade off in selecting a proper r .

Now, Theorem 1 quantifies the amount of querying budget that is sufficient to obtain arbitrary detection probability by choosing the optimal r^* in the sense of the lower bound in (6). We provide the proof in Section V.

Theorem 1: Using **SB-Q**(r^*), where

$$r^* = \left\lceil 1 + \frac{(1-p)\log K}{2e\log(d-1)} \right\rceil,$$

for any given $0 < \delta < 1$, the detection probability under d -regular tree is at least $1 - \delta$, if

$$K \geq \frac{4(d-1)/(d-2)(2/\delta)}{p^2 \log(2/\delta)}. \quad (7)$$

This results indicates that if untruthfulness probability is such that $p > 1/2$, then we need $1/p^2$ times more budget of querying to satisfy the same target probability compared to the case of $p = 1$, as analyzed in [19], *e.g.*, about 4 times when $p \simeq 1/2$. To illustrate, consider $p = 0.6$ and $d = 3$, where we need $K \geq 684$ to achieve at least 95% accuracy of detection.

IV. INTERACTIVE QUERYING WITH DIRECTION

In this section, we study the case of interactive querying with direction. Recall that a source finding algorithm for interactive queries consists of the following steps: We first need to appropriately choose the repetition count r and the initial node v_I to ask the identity question. If she is not the source then ask the direction questions r times. From the answers of querying, the querier chooses a next node to perform the same procedure.

A. Ideal Algorithm and Challenges

Two key components for high detection probability are the choice of the repetition count and a smart policy which selects the next queree based on the answer sample for the direction questions.

Let $\mathcal{P}(v_I)$ be a set of all policies, each of which provides a rule of choosing a next queree at each querying step, when the initial queree is v_I . Once a policy $P \in \mathcal{P}(v_I)$ and r are chosen, the estimated node \hat{v} is determined, *i.e.*, the node who reveals itself as the rumor source for the identity question, or the last queree, otherwise. Then, it is natural to choose r and P so as to solve the following optimization:

$$\text{OPT-I: } \max_{1 \leq r \leq K} \max_{v_I \in V_N} \max_{P \in \mathcal{P}(v_I)} \mathbb{P}[\hat{v} = v_1 | v_I]. \quad (8)$$

We now explain the technical challenges in solving **OPT-I**. We first introduce some notations for expositional convenience as well as our analytical results later. Let w_i be the i -th queree for $1 \leq i \leq K/r$ ⁵, and let $Z_r := Z_r(P) = (z_1, z_2, \dots, z_{K/r})$ be the sequence of answers for the identity questions to each queree for a given policy P , where $z_i \in \{\text{no}, \text{yes}\}$ for the queree w_i . We also let $D_r^i := D_r^i(q) = [y_1, y_2, \dots, y_d]$ be the answer vector for the queree i , where $0 \leq y_j \leq r$ that represents the number of “designations” to j -th neighbor ($1 \leq j \leq d$) as w_i ’s parent.

As in the simple batch querying, it is important to obtain an analytical form of the solution of the following problem, to choose the right r : for a fixed $1 \leq r \leq K$:

$$\text{SUB-OPT-I: } \max_{v_I \in V_N} \left(\max_{P \in \mathcal{P}(v_I)} \mathbb{P}[\hat{v} = v_1 | v_I] \right). \quad (9)$$

To solve **SUB-OPT-I**, consider the probability $\mathbb{P}[\hat{v} = v_1 | v_I]$ in (9) for a given v_I . First, it is pretty challenging to find an optimal policy P , because P ’s action at each i -th queree can

⁵In our model, i can be strictly less than K/r if there exists i such that i -th queree answers “yes” for the identity question.

Algorithm 2: ID-Q(r): r : Repetition Count

Input: Diffusion snapshot G_N , querying budget K , degree d , and truth probabilities $q > 1/d$

Output: Estimated rumor source \hat{v}

```
1 Calculate the rumor centrality  $R(v, G_N)$  for all  $v \in G_N$ 
  as in [1] and let  $s \leftarrow \arg \max_{v \in V_N} R(v, G_N)$ ;
2 while do
3   if  $s = v_1$  then
4      $K \leftarrow K - 1$ ;
5     Break ;
6   else
7     if  $K \geq r$  then
8       Count the number of “designations” (i.e., She
        has spread the rumor to me) for the direction
        question among  $s$ 's neighbors, and choose the
        largest counted node with a random tie
        breaking;
9       Set such chosen node by  $s$ ;
10       $K \leftarrow K - r$ ;
11 Return  $\hat{v} = s$ ;
```

be considered as a mapping \mathcal{F}_i that uses the entire history of the querees and their answers:

$$\mathcal{F}_i : \{D_r^1, D_r^2, \dots, D_r^{i-1}; w_1, \dots, w_{i-1}\} \rightarrow V_N, \quad (10)$$

for each i . As an approximation, it is natural consider the mapping $\mathcal{F}_i : (D_r^{i-1}, w_{i-1}) \rightarrow V_N$, *i.e.*, the next queree is determined only by the information at the moment. Even under this approximation, it also remains to estimate the true parent node of the queree, using her answers. To handle this issue, we may consider the MLE to estimate the true parent node, *i.e.*, $\max_{v \in nb(w_i)} \mathbb{P}[G_N, D_r^{i-1} | v = \text{parent}(w_i)]$, where $nb(w_i)$ is the set of the neighbors of w_i . However, this is also not easy to analyze, because for some v the probability that it is a true parent requires to compute the probability that the true source is located in v 's subtree which does not contain w_i .

Thus, we propose a heuristic algorithm that is designed to produce an approximate solution of **OPT-I**. The key of our approximate algorithm is to choose the policy that allows us to analytically compute the detection probability for a given r so as to compute r easily, yet its performance is close to that of **OPT-I**.

B. Algorithm based on Majority Rule

The key idea our algorithm is that we simply apply a *majority-based rule*, as will be clarified soon. We first formally describe our algorithm, called **ID-Q(r)** and explain how it operates and its rationale. Again, **ID-Q(r)** is parameterized by the repetition count r : we first calculate the rumor centrality of all nodes in G_N (Line 1), where s is set to be the rumor center. Then, we query the identity question to the rumor center s (Lines 3-5). If she is not the source then ask the direction questions r times, and count the number of “designations” for its neighbor (Lines 7-8). Then, we choose the largest

counted node with a random tie breaking and repeat the same procedure. The algorithm stops when there is a node which reveals itself as the rumor source within K queries, otherwise, it outputs the last queried node as the estimator.

In selecting a parent node of the target queree, instead of the exact calculation of MLE, a simple majority voting is used by selecting the node with the highest number of designations, motivated by the fact that when $q > 1/d$, such designation sample can provide a good clue of who is the true parent. Using this idea, we are able to have an approximate, but closed form solution of **SUB-OPT-I**, which allows us to compute the best r^* that maximizes the detection probability under such an approximation, as will be given in the next section.

C. Detection Performance

We now provide analytical results on the detection performance of **ID-Q(r)**. We first start by presenting the lower bound of the detection probability for a given repetition count r in Proposition 3

Proposition 3: For d -regular trees ($d \geq 3$), a snapshot G_N , a given budget K , our estimator $\hat{v}(G_N, r)$ from **ID-Q(r)** has the detection probability lower-bounded by:

$$\lim_{N \rightarrow \infty} \mathbb{P}[\hat{v}(G_N, r) = v_1] \geq 1 - c \cdot \exp \left[-2(g_d(r, q))^3 \left(\frac{K}{r+1} \right) \log \left(\frac{K}{r+1} \right) \right], \quad (11)$$

where $g_d(r, q) := 1 - e^{-\frac{r(d-1)}{3d(1-q)}}$ and $c = (8d+1)/d$.

The proof is presented in Section V. The term $g_d(r, q)$ in (11) is the probability that the queree reveals the true parent for given r and q . Next, Theorem 2 quantifies the amount of querying budget that is sufficient to obtain arbitrary detection probability by choosing the optimal r^* in the sense of the lower bound in (11).

Theorem 2: Using **ID-Q(r^*)**, where

$$r^* = \left\lceil 1 + \frac{2d(1-q)^2 \log \log K}{3(d-1)} \right\rceil,$$

for any given $0 < \delta < 1$, the detection probability under d -regular tree is at least $1 - \delta$, if

$$K \geq \frac{(2d-3)/d(\log(7/\delta))}{q^3 \log(\log(7/\delta))}. \quad (12)$$

The proof is presented in Section V. This theorem indicates that if querees are truthful with probability $q > 1/d$, we need $1/q^3$ times more querying budget than the case without untruthfulness, *i.e.*, $q = 1$, *e.g.*, 8 times when $q = 1/2$. As an example, suppose $q = 0.4$ and $d = 3$, where we need $K \geq 51$ to achieve at least 95% accuracy of detection.

It is expected that the querying with direction helps and thus requires less budget than simple batch querying. Our contribution lies in quantifying this difference: for small δ , with respect to the scaling of $1/\delta$, the amount of querying is asymptotically reduced from $1/\delta$ to $\log(1/\delta)$, which should be significant especially when high detection quality is necessary.

V. PROOFS OF RESULTS

A. Proof of Proposition 2

First, for a given r , we introduce the notation V_l , which is equivalent to C_r , where the hop distance l is given in Line 3 of **SB-Q**(r). This is for presentational simplicity due to a complex form of l . Also for notational simplicity, we simply use $\mathbb{P}[\hat{v} = v_1]$ to refer to $\lim_{N \rightarrow \infty} \mathbb{P}[\hat{v}(G_N, r) = v_1]$ in the proof section. Then, the detection probability is expressed as the product of the three terms:

$$\begin{aligned} \mathbb{P}[\hat{v} = v_1] &= \mathbb{P}[v_1 \in V_l] \times \mathbb{P}[\hat{v} = v_1 | v_1 \in V_l] \\ &= \mathbb{P}[v_1 \in V_l] \times \mathbb{P}[v_1 \in \hat{V} | v_1 \in V_l] \\ &\quad \times \mathbb{P}[v_1 = v_{LRC} | v_1 \in \hat{V}], \end{aligned} \quad (13)$$

where \hat{V} is the filtered candidate set (Lines 4-5 of **SB-Q**(r)) and v_{LRC} is the node in \hat{V} that has the highest rumor centrality, where LRC means the local rumor center. We will drive the lower bounds of the first, second, and the third terms of RHS of (13). The first term of RHS of (13) is bounded by

$$\mathbb{P}[v_1 \in V_l] \geq 1 - c \cdot e^{-(l/2)\log l}, \quad (14)$$

where the constant $c = 7(d+1)/d$ from Corollary 2 of [19]. The second and the third terms are handled by the following two lemmas, whose proofs are will be provided in our technical report [21]:

Lemma 1: When $p > 1/2$,

$$\mathbb{P}[v_1 \in \hat{V} | v_1 \in V_l] \geq p + (1-p)(1 - e^{-p^2 \log r}).$$

Lemma 2: When $d \geq 3$ and $p > 1/2$,

$$\mathbb{P}[v_1 = v_{LRC} | v_1 \in \hat{V}] \geq 1 - e^{-p^2 r \log r}.$$

Then a simple algebra gives us the result that the product of two lower bounds in Lemmas 1 and 2 is also lower-bounded by $1 - e^{-p^2 \log r} \geq 1 - \frac{1-p}{r+1} = \frac{r+p}{r+1}$, where we use the fact that $e^{-p^2 \log r} \leq (1-p)/(r+1)$. Merging this lower-bound with the lower-bound in (14) where we plug in $l = \frac{\log(\frac{K(d-2)}{rd} + 2)}{\log(d-1)}$, the result follows. This completes the proof.

B. Proof of Theorem 1

We first note that the event that our algorithm does not detect the source to derive the detection error probability is the union of the following two disjoint events: $E_1 := \{d(v_{RC}, v_1) > l\}$ and $E_2 := \{\hat{v} \neq v_1 | d(v_{RC}, v_1) \leq l\}$, where E_1 is the event that the source is not in the candidate set V_l and E_2 is the event that our estimator fails to detect the source conditioned that the source is in V_l . Then, from Lemmas 1 and 2, we get:

$$\mathbb{P}[\hat{v} \neq v_1] \leq (1-p)e^{-p^2 r \log r} + c \cdot e^{-\frac{l}{2} \log l}, \quad (15)$$

where the constant c is the same as that in (14). Now, we first put $l = \frac{\log(\frac{K(d-2)}{rd} + 2)}{\log(d-1)}$ into (15) and obtained the upper-bound of (15), expressed as a function of r , for a given p , and the constant c . Then, we take r^* in the theorem statement which

is derived in [21] and put it to the obtained upper-bound which is expressed as a function of K , as follows:

$$\begin{aligned} \mathbb{P}[\hat{v} \neq v_1] &\leq (1-p)p^2 e^{-\log K \log(\log K)} + cp^2 e^{-\frac{\log K}{2} \log(\log K)} \\ &\leq (1-p)p^2 e^{-\frac{\log K}{2} \log(\log K)} + cp^2 e^{-\frac{\log K}{2} \log(\log K)} \\ &\leq c_1 p^2 e^{-\frac{\log K}{2} \log(\log K)}, \end{aligned} \quad (16)$$

where $c_1 = c+1$. If we set $\delta \geq c_1 p^2 e^{-\frac{\log K}{2} \log(\log K)}$, we find the value of K such that its assignment to (16) produces the error probability δ , and we get the desired lower-bound of K as in the theorem statement. This completes the proof.

C. Proof of Proposition 3

As a first step, we will obtain the upper bound of the error probability when $r = 1$ i.e., only one direction question is used. After obtaining this, we can easily extend the result for general r which will be provided in later. By similar approach in the proof of Theorem 1, we define two error events such as $E_1 := \{d(v_{RC}, v_1) > K/2\}$ and $E_2 := \{\hat{v} \neq v_1 | d(v_{RC}, v_1) \leq K/2\}$ which are disjoint. From Lemma 2, we have $\mathbb{P}[E_1] \leq c \cdot e^{-(K/4)\log(K/2)}$ since we use additional direction query with identity question. Next, by conditioning on the distance $d(v_{RC}, v_1) = i$, the probability for the event E_2 is given by

$$\mathbb{P}[E_2] = \sum_{i=1}^{K/2} \mathbb{P}[\hat{v} \neq v_1 | d(v_{RC}, v_1) = i] \mathbb{P}[d(v_{RC}, v_1) = i].$$

We first obtain $\mathbb{P}[\hat{v} \neq v_1 | d(v_{RC}, v_1) = i]$ when the total budget is K . To do this, let X_j be the random variable which takes $+1$ if the answer is correct at j -th direction query by the querier and takes -1 , otherwise. Then, the error event is occurred when $\sum_{j=1}^{K/2} X_j < i$ for all $1 \leq j \leq K/2$ because the querier can not meet the source for the case. Hence, for a given $q > 1/d$, we have

$$\begin{aligned} \mathbb{P}[\hat{v} \neq v_1 | d(v_{RC}, v_1) = i] &\leq \sum_{j=0}^{i-1} \binom{K/2}{j} q^j (1-q)^{K/2-j} = I_{1-q}(K/2 - i, i) \\ &= (K/2 - i) \binom{K/2}{i-1} \int_0^{1-q} t^{K/2-i} (1-t)^{i-1} dt \\ &\stackrel{(a)}{\leq} \exp\left(-\frac{2((K/2)q - (i-1))^3}{K/2}\right) \leq \exp(-2q^3(K/2)^2), \end{aligned}$$

where (a) is due to the Hoeffding bound for the regularized incomplete beta function $I_{1-q}(K/2 - i, i)$ when $q > 1/d$. Hence, from the fact that $\mathbb{P}[d(v_{RC}, v_1) \leq K/2] \leq 1$, we obtain $\mathbb{P}[E_2] \leq (K/2)e^{-2q^3(K/2)^2}$. By combining the probabilities $\mathbb{P}[E_1]$ and $\mathbb{P}[E_2]$, the total probability of error is bounded by

$$\begin{aligned} \mathbb{P}[\hat{v} \neq v_1] &\leq c \cdot e^{-(K/4)\log(K/2)} + (K/2)e^{-2q^3(K/2)^2} \\ &\leq (c+1)(K/2)e^{-2q^3(K/2)^2} \stackrel{(a)}{\leq} c_1 e^{-2q^3(K/2)\log(K/2)}, \end{aligned} \quad (17)$$

where $c_1 = c+1$ and the inequality (a) is from the fact that $(K/2)e^{-(K/2)} \leq K/2 = e^{\log(K/2)}$. Hence, we conclude

the result for $r = 1$. Based on this, we extend (17) for general r . First, consider the total number of direction queries is $r \geq 1$ for each queriee and let $Y_i^1(v)$ be the random variable which takes +1 for the i -th query when the true parent node is designated by the queriee v with probability q and let $Y_i^j(v)$ be the random variable which takes +1 for the i -th query when one of other neighbor nodes $2 \leq j \leq d$ is designated by v with probability $(1 - q)/(d - 1)$. Define $Z_j(v) := \sum_{i=1}^r Y_i^j(v)$ be the total number of designations by the node v for the j -th neighbor ($1 \leq j \leq d$). Then, we need to find $\mathbb{P}[Z_1(v) > Z_j(v), \forall j]$ which is the probability that the true parent is the node with maximum designations by queriee v . This probability is handled by the following lemma whose proof is given in [21].

Lemma 3: If $q > 1/d$ then

$$\mathbb{P}[Z_1(v) > Z_j(v), \forall j] \geq 1 - e^{-\frac{r(d-1)}{3d(1-q)}}.$$

From this result and (17), we obtain

$$\mathbb{P}[\hat{v} \neq v_1] \leq ce^{-2(g_d(r,q))^3(K/(r+1)) \log(K/(r+1))}, \quad (18)$$

where $c = (8d + 1)/d$ and $g_d(r, q) := 1 - e^{-\frac{r(d-1)}{3d(1-q)}}$. Hence, we complete the proof of Proposition 3.

D. Proof of Theorem 2

By r^* in the theorem statement which is derived in [21] and put it to the obtained upper-bound in (18) then

$$\begin{aligned} \mathbb{P}[\hat{v} \neq v_1] &\leq ce^{-2\left(1 - e^{-\frac{r(d-1)}{3d(1-q)}}\right)^3 (K/(r+1)) \log(K/(r+1))} \\ &\leq ce^{-2\left(1 - e^{-(1-q) \log \log K}\right)^3 (K/(r^*+1)) \log(K/(r^*+1))} \\ &\stackrel{(a)}{\leq} ce^{-2\left(1 - (\log K)^{-(1-q)}\right)^3 (K/(r^*+1)) \log(K/(r^*+1))} \\ &\stackrel{(b)}{\leq} ce^{-2q^3 K^2 \log(K/(r^*+1))} \leq ce^{-2q^3 K \log K}, \quad (19) \end{aligned}$$

where the inequality (a) is from the fact that $1 - x \leq e^{-x}$ for $0 \leq x \leq 1$ and (b) is due to the fact that $q > 1/d$ with $K \log(K/(r^* + 1)) > \log K$. Let $\delta \geq ce^{-2q^3 K \log K}$ then, we obtain the value of K which produces the error probability δ in (19) and we obtain the desired lower-bound of K as in the theorem statement. This completes the proof.

VI. SIMULATION RESULTS

In this section, we will provide simulation results of our two proposed algorithms over three types of graph topologies: (i) regular trees, (ii) two random graphs, and (iii) a Facebook graph. We propagate a rumor from a randomly chosen source up to 400 infected nodes, and plot the detection probability from 200 iterations.

Regular trees. We use d -regular tree with $d = 3$, where we compare three algorithms for both simple batch querying (SB-Q) and interactive querying with direction (ID-Q): our algorithms, denoted by L -hop multi query, MLE single query, and MLE multi query. MLE single query and MLE multi query are the algorithms that we use $r = 1$ and $r = r^*$

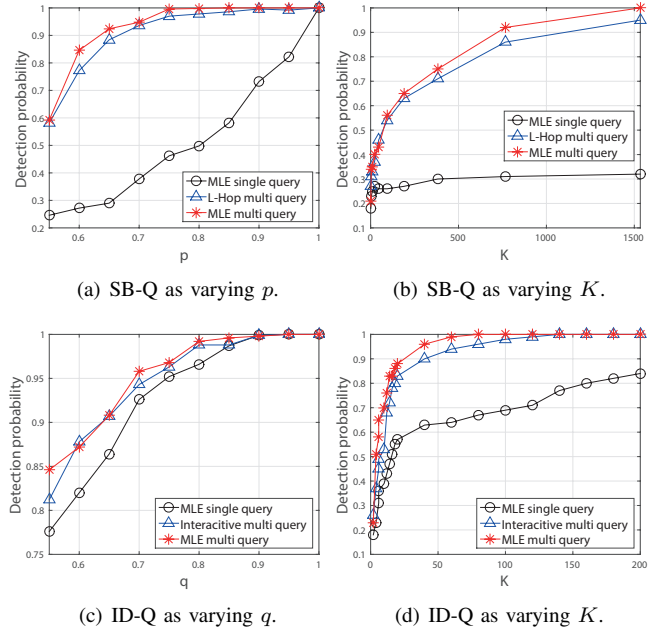


Fig. 2. Detection probabilities for d -regular trees ($d = 3$) in batch querying ((a),(b)) and interactive querying with direction ((c),(d)), respectively.

(as described in Theorems 1 and 2), respectively, but for those fixed r , MLE based estimation algorithms are used as discussed in Sections III-A and IV-A. Although MLE multi query is not theoretically optimal, we believe that it is close to optimal, providing the information on how closely our algorithms perform compared to optimal ones. Fig. 2(a) shows the detection probabilities for simple batch querying, as the truth probability p varies from 0.55 to 1 when $K = 766$ (corresponding to the number of nodes within 8 hop distance). As expected, the probability increase as p increases and we see that if $p = 0.7$ then the detection probability is about 40% for the MLE single query whereas above 90% for multi query. In Fig. 2(b), we vary the query budget for $p = 0.6$. For multi querying, 1000 queries are enough to achieve the detection probability is at least 90% however, it is not beyond 50% even for $K = 1500$ for the single querying even for MLE, implying that just selecting a large number of candidate nodes is not enough for untruthful users, and a certain procedure of learning in presence of untruthfulness such as multi querying becomes essential. Figs. 2(c) and 2(d) show the similar kind of plots for interactive querying as p and K vary, respectively. We observe that when the detection probability is above 99% even for MLE single query if $q > 0.9$. However, for $q = 0.4$ in Fig. 2(d), we see that the detection probability is below 90% when $K = 200$ whereas those of both multi querying schemes are almost one, showing the power of interactiveness in querying.

Random graphs. We consider *Erdős-Rényi* (ER) and scale-free (SF) graphs. In the ER graph, we choose its parameter so that the average degree by 4 for 2000 nodes. In the SF graph, we choose the parameter so that the average ratio of edges to nodes by 1.5 for 2000 nodes. It is known that obtaining MLE is hard for the graphs with cycles, which is $\#P$ -complete.

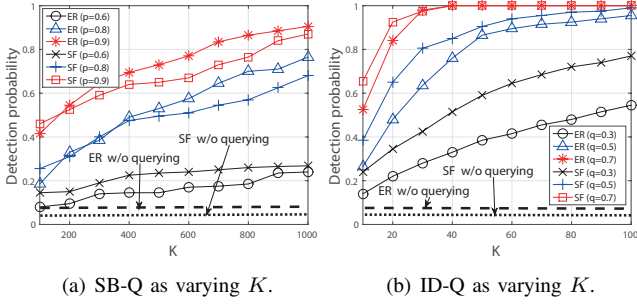


Fig. 3. Detection probabilities for ER and SF graphs. (Without querying is the detection probability by BFS estimator.)

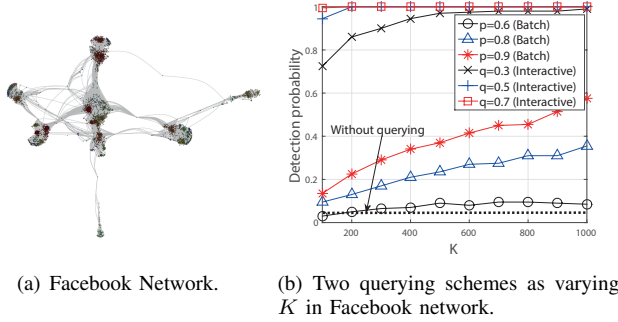


Fig. 4. Detection probabilities for Facebook network. (Without querying is the detection probability by BFS estimator.)

Due to this reason, we first construct a diffusion tree from the Breadth-First Search (BFS) as used in [1]: Let σ_v be the infection sequence of the BFS ordering of the nodes in the given graph, then we estimate the source v_{bfs} that solves the following:

$$v_{\text{bfs}} = \arg \max_{v \in G_N} \mathbb{P}(\sigma_v | v) R(v, T_b(v)), \quad (20)$$

where $T_b(v)$ is a BFS tree rooted at v and the rumor spreads along it. Then, by using those selected nodes, we perform our algorithms with querying. Figs. 3(a) and 3(b) show the detection probabilities with varying K for batch and interactive querying, where we observe similar trends to those in the regular trees. We see that only about 50 questions need to be asked to achieve 99% detection probability when $q = 0.7$ for the interactive querying scheme.

Real world graph. Finally, we show the results for a Facebook network as depicted in Fig. 4(a). We use the Facebook ego network in [22] which is an undirected graph consisting of 4039 nodes and 88234 edges where each edge corresponds to a social relationship (called FriendList) and the diameter is 8 hops. We perform the same algorithm used for random graphs based on the BFS heuristic and show the results in Fig. 4(b). The results show that how fast the detection probabilities goes to one as K increases. For example, the interactive querying requires about 200 queries to achieve almost one detection probability when $q > 0.5$.

VII. CONCLUSION

In this paper, we have considered the querying framework with untruthful answers in rumor source detection. We have provided some theoretical performance guarantees when the underlying network has regular tree structure. We obtain how much query budget is required for two querying types to achieve the target probability when the truth probabilities are homogeneous in the queries. We perform various simulations based on these algorithms. As future works, we will consider the hidden heterogeneous truth probabilities in answers for both querying scenarios.

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