On the Economic Effects of User-oriented Delayed Wi-Fi Offloading

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***Abstract*—Both users and mobile network providers increasingly suffer from explosive growth of mobile traffic. We study so-called delayed Wi-Fi offloading that has been recently proposed as a low-cost solution of alleviating mobile data explosion. Delayed Wi-Fi offloading is a technology that offloads traffic from cellular to Wi-Fi by persuading users into delaying their delay-tolerant traffic and thus actively exploiting users' chance to meet Wi-Fi. In this paper, we study the economic effects of such user-oriented delayed Wi-Fi offloading in the monopoly market as well as the market with two providers. In the monopoly market, we model a two-stage game, where the provider selects an offloading price for which users are the price-takers. In the market with two providers, we consider a situation that either of providers, say A, launches a delayed Wi-Fi offloading service as a separate service from the original cellular service, thus allowing the users in another provider, say B, to subscribe to the offloading service from A, but with some incurring switching cost. In both markets, we study how the equilibrium offloading price changes depending on other system parameters, e.g., cellular cost, Wi-Fi density, and the number of subscribers, by computing the Nash equilibriums and providing extensive numerical results for various parameter changes, which gives us useful insights into how economically viable user-oriented delayed Wi-Fi offloading is.**

I. Introduction

A. Motivation and Summary

The mobile data traffic is explosively increasing as smart phone users increase and user devices experience unprecedented technological advances. According to the popularly-cited Cisco report [1], the total mobile data traffic demand is expected to be 6.3 exabytes per month in the year 2015, which is 26.25 times larger than the year 2010's total mobile data traffic demands. To tackle the mobile data explosion, people consider an option of offloading via Wi-Fi or femtocell [2]–[7]. In particular, Wi-Fi offloading, where users transmit their traffic via Wi-Fi AP instead of 3G/4G cellular, seems to be one of the promising solutions because Wi-Fi AP is relatively of low cost and has also been already widely deployed by mobile providers as well as individual users.

To maximize the effect of Wi-Fi offloading's cellular traffic reduction, a notion of *delayed* Wi-Fi offloading (or simply delayed offloading) has been proposed [4], [5]. Their example use-cases include the one that a user specifies diverse delay deadlines for the delivery of their delay-tolerant data and only Wi-Fi connection is used for the data delivery before the deadline while she moves, and 3G/4G is used just when time is close to the specified deadlines. The effect of delayed offloading has experimentally been verified in the recent researches, see e.g., [4], [5] show that about up-to 80% of cellular data traffic can be offloaded to Wi-Fi. Viability of delayed offloading is also supported by the fact that both there exists non-negligible portion of delay-tolerant data in reality and users may be willing to delay their data, once proper economic incentives are provided, e.g., discount of service fee. A recent survey [8] positively supports such a "hope", reporting that users are indeed willing to wait 5 minutes (for YouTube videos) to 48 hours (for software upgrades).

However, despite the experimental validation of the effect of the delayed offloading as well as its survey-based viability, it is still questionable whether such a new service would be actively adopted in practice or not. Major concerns include (i) reduction of cellular usage and additional incentive to users may decrease cellular provider's revenue and (ii) users' unhappiness due to delaying delivery completion time may beat increasing utility stemming from the service fee discount, so that users may be reluctant to use this delayed offloading service. This motivates our study in this paper that models the economic relation between a single or a multiple of providers and users, and quantifies the economic effect of delayed Wi-Fi offloading.

The main contributions of this paper are as follows:

1) We first consider a monopoly market, i.e., a single provider. We propose an economic model for a delayed Wi-Fi offloading service (offloading service in short) with delay sensitivity. Users are sensitive to delay and their utility decreases as delay increases. This utility reduction was ignored in prior work, which seems to be the crucial part in the economic analysis of offloading due to the opportunistic Wi-Fi AP meetings.
2) We prove that offloading service is beneficial for both providers and users. We show that as the operational cost for a cellular service increases, more Wi-Fi APs are installed, or longer deadline that a user can set for delaying its delay-tolerant data, the provider is more willing to launch a offloading service with lower price to increase its revenue.
3) We next consider a market with two providers where provider A who offers the both offloading and cellular services and provider B offers only cellular service. In this duopoly market, the provider A chooses lower offloading

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price to attract the provider B's users to use offloading service when the switching cost (an incurring cost due to provider change) is smaller than a certain threshold cost, whereas the provider A does not attract provider B's users and sets its offloading price as the same offloading price as in monopoly market when the switching cost exceeds the threshold cost.

B. Related Work

We first summarize prior work which we think are directly related to delayed Wi-Fi offloading. First, in terms of the offloading effect of delayed Wi-Fi offloading, K. Lee *et al.* [4] and A. Balasubramanian *et al.* [5] experimentally showed that users' delayed transmission with diverse given deadlines indeed enables a huge portion of data to be offloaded via Wi-Fi. There exists a work which studied how incentives can be given to users. X. Zhuo *et al.* [6], [9] proposed an auction-based scheme; each user sends a bid with its delay tolerance and intended incentive for which the mobile network provider selects a winner. The related work on economic analysis of delayed Wi-Fi offloading includes that by J. Lee *et al.* [10], which provided a game-based economic analysis of users' utility and a monopoly provider's revenue for delayed Wi-Fi offloading with focus on the impact of pricing schemes. This paper's major difference from [10] lies in they focus only on a monopoly provider, and more importantly they ignore users' delay sensitivity, i.e., utility reduction due to delay of data delivery was ignored. C. Joe-Wong *et al.* [11] studied how and when users adopt a service given two options: a base technology (i.e., only cellular service) and a new bundled service (i.e., cellular + Wi-Fi offloading), assuming that users have heterogeneous valuation for each technology. This is close to the analysis of this paper and [11] for the case of monopoly provider, but with more focus on the technology adoption and the dynamic behaviors of subscribers.

Other related work includes TUBE [12] which is a framework of time dependent pricing, which allows users to delay their data to off-peak times, where the authors provided a prototype implementation in conjunction with experiments with real users. This work of [12] showed a feasibility of alleviating the amount of peak-time traffic through an appropriate pricing scheme and discussed the necessary system-side issues and their solutions. D. Wei *et al.* [13], I. George *et al.* [14], and G. Lin *et al.* [15] proposed an auction-based incentive framework that enables third-party resource owners (i.e., personalized Wi-Fi owners) to share their devices.

II. System Model

A. Network and Service Model

Offloading service. A mobile network operator (MNO) (or simply provider), which has already provided a *cellular service*, now launches a new service, called *delayed Wi-Fi offloading service* (or simply offloading service). For offloading service, the MNO may install new additional Wi-Fi APs in various hot spots. Whenever a user has data to download, the user subscribing to the offloading service specifies a deadline d in advance, where the provider is responsible for providing a facility (e.g., offloading server module which manages user-specified deadlines) that enables the user to download the data only using Wi-Fi connections before the deadline d expires and finally use cellular connections only when time is close to the deadline. We assume that users are always in the coverage of cellular service, but offloading service is just possible when they are under Wi-Fi coverage.

Pricing and mobility. We assume usage-based pricing for both services [1]. Let p and q be the prices per a unit data volume for cellular and offloading services, respectively, where $q < p$. Let b $(< p)$ be the operational cost of transporting a unit data volume through the cellular service, and assume that no cost is incurred for data transmission over Wi-Fi. While users move, the chances to meet Wi-Fi APs differ depending on their mobility patterns and the number and the locations of installed Wi-Fi APs. In our model, we capture it by the inter-AP meeting time, which is assumed to be exponentially distributed with a parameter λ. We focus on the non-trivial case, called *sparse Wi-Fi regime* that (i) the mean Wi-Fi inter-meeting time $1/\lambda$) is much longer than the content transmission time via cellular or Wi-Fi network, and (ii) $1/\lambda$ is smaller than d (i.e., $\lambda d < 1$), meaning that on average, data delivery is not possible only through transmission over Wi-Fi [2].

Assumptions. We make a few assumptions as in what follows: First, deadline is user- and application-dependent, but in this paper they are assumed to be homogeneous across users and applications. Homogeneous deadline may imply that we consider a scenario that all offloading users subscribe to the offloading service for a single content type, i.e., offloading for downloading movie contents. Second, whenever data is transmitted through Wi-Fi, the AP contact time (AP connection time during which a user is in its coverage) is large enough to finish the data delivery using a single AP contact. The assumptions have been made mainly for tractable analysis, since modeling the economic interaction between users and providers are the major ones that we focus on in this paper, which are highly complex, as will be described in this paper. One may regard our homogeneous assumption as the averaged parameters before other parts such as user-provider coupling are analyzed.

In Sections II-B and II-C, we describe the model for users and providers with implicit assumption that we focus on the case of a single provider. We will describe the minor difference in models for the case of two providers in Section II-D.

B. Users

Utility function. There are N users and we are interested in how happy each user is about the delay of a data delivery. To that end, each user is modeled to have her own delay sensitivity θ, which is uniformly random over the interval $[0, 1]$. The utility of a user with with delay sensitivity θ who finishes

[1] As of spring of 2012, the two largest U.S. wireless providers, AT&T and Verizon Wireless had announced their mobile data usage policies, effectively imposing usage-based pricing [12], [16].

[2] This is reasonable, because MNO's major business seems focused on the cellular service, using Wi-Fi as a supplemental data delivery network. Thus, Wi-Fi tends to be installed only at the hot spots.

data delivery at time t is modeled as:

$$U_\theta(t) = \begin{cases} -\frac{1}{d}\theta t + 1 & \text{if} \quad 0 \le t < d, \\ 1-\theta & \text{if} \quad t \ge d, \end{cases} \tag{1}$$

meaning that the utility without any deliver delay is 1, decreasing as more delay is incurred, and for the delivery time larger than the deadline d, the utility is kept at some minimum value. Here, we see that larger θ implies more delay sensitivity, meaning they are more reluctant to delay their data. Note that under our assumption of sparse Wi-Fi regime, we ignore the data transmission delay. Fig. 1 illustrates the shape of utility functions for various θ values when $d = 40$. Our choice of utility function in (1) with a piece-wise linear form is due to our intention of both making our analysis tractable and reflecting delay sensitivity in the functions. Since $U_\theta(t) \le 1$ for all t, we should have the condition that $p \le 1$ (i.e., *user rationality condition*).

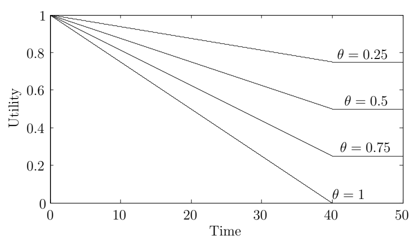


Fig. 1. Utility function for $\theta = 0.25, 0.5, 0.75, 1$ where $d = 40$, where x-axis represents data delivery time.

Strategies. Each user is rational, and has two strategies, P or I: (i) P(atient), i.e., subscribing to both cellular and offloading services and (ii) I(mpatient), i.e., subscribing to only cellular service.

(i) *Impatient.* We call the user with cellular-only service, as impatient user, and its net-utility is:

$$\pi_\theta(I) = U_\theta(0) - p = 1 - p, \tag{2}$$

where $U_\theta(0)$ refers to the utility without any delay and p is the price for using cellular service. Recall that $p \le 1$.

(ii) *Patient.* We call the user who additionally uses offloading service, as patient user, and its (expected) net-utility is:

$$\pi_\theta(P) = \int_0^d \lambda e^{-\lambda t}\Big(-\frac{1}{d}\theta t + 1 - q\Big)\mathrm{d}t + e^{-\lambda d}\Big((1-\theta) - q\Big) = \theta\left(\frac{e^{-\lambda d} - 1}{\lambda d}\right) - q + 1, \tag{3}$$

where the net-utility is the sum of the cases when users meet a Wi-Fi AP before and after the deadline. Note that $e^{-\lambda d}$ is the probability that a user does not meet Wi-Fi AP till time d, and $(1-\theta)$ is the utility when the user finished content download after the deadline d (see (1)).

C. Provider

The Provider is also rational and chooses an offloading service fee q as a strategy. Depending on the choice of q, her (expected) revenue is:

$$R(q) = N_I(q)(p - b) + N_P(q)(q - be^{-\lambda d}) \tag{4}$$

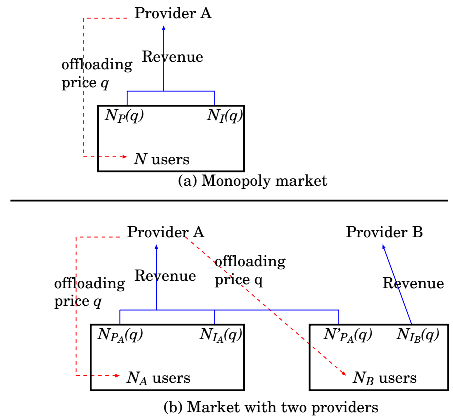


Fig. 2. Two market models in this paper: (a) monopoly market and (b) market with two providers.

where $N_I(q)$ and $N_P(q)$ ($N_I(q) + N_P(q) = N$) are the expected numbers of impatient and patient users, respectively. Note that they are the functions of q, because we later see that users respond to the choice of q by the provider, which determines $N_I(q)$ and $N_P(q)$ (see Fig. 2(a)). Note that $be^{-\lambda d}$ corresponds to the volume of data that are not offloaded and thus transmitted through the cellular network. Clearly, the price for offloading service should exceed the marginal price ($q > be^{-\lambda d}$), and similarly $p > b$, which we call *provider rationality condition*. Then, the objective of the provider is to choose the revenue maximizing $q^\star$:

$$q^\star = \underset{q}{\text{argmax}} \quad R(q) \qquad \text{subject to} \quad be^{-\lambda d} < q < p. \tag{5}$$

D. Market with Two Providers

We will also consider the case of a market with two providers, where there exist two providers, named A and B, with N_A and N_B users. Let $N_A = kN_B$, where $k > 0$ quantifies the difference of the user shares of two providers, i.e., if $k > 1$, the provider A has larger market share. We are interested in what happens when only a single provider launches the offloading service, say provider A in our case. In this case, we consider a market with *service separation*, meaning that a user who is a cellular service user of provider B can subscribe to the offloading service of provider A. To reflect user's burden of using different providers for offloading service, we introduce a notion of provider switching cost c. Since the provider B does not offer offloading service, we just use λ (rather than λ_A) to refer to provider A's Wi-Fi AP meeting rate, for notational simplicity.

The model of this market with two providers with service separation is the same as that in monopoly market, except for the utility of a provider B's user who subscribes to the offloading service by the provider A mainly due to the switching cost c (although the analysis becomes much more complex as seen in Section IV). In this case, such a provider B user's net-utility, which we denote by $\pi'_\theta(P_A)$, is given by:

$$\pi'_\theta(P_A) = \theta\frac{(e^{-\lambda d} - 1)}{\lambda d} - (q + c) + 1, \tag{6}$$

where P_A corresponds to the strategy that chooses to be patient with provider A's offloading service.

To clearly express the number of users subscribing to different services, we abuse notations, and use the followings: Let $N_{I_A}(q)$ and $N_{P_A}(q)$ be the number of provider A users who are impatient and patient, respectively. Similarly, we use $N_{I_B}(q)$ to refer to the number of provider B users who are impatient, and to represent the number of provider B users who subscribe to the offloading service of provider A, we use the notation N'_{P_A} (see Fig. 2 for a model illustration). Then, the provider A's revenue is expressed by:

$$R(q) = N_{I_A}(q)(p-b) + (N_{P_A}(q) + N'_{P_A}(q))(q - be^{-\lambda d}), \quad (7)$$

where term $N'_{P_A}(q)$ is the additional one, differing from the revenue in the monopoly market, as in (4). The provider B's revenues is expressed by:

$$R(q) = N_{I_B}(q)(p-b) - N'_{P_A}(q)(q - be^{-\lambda d}). \quad (8)$$

III. Analysis: Monopoly Market

We first study the monopoly market. To model it, given the system parameters p, b, N, and λ, we consider a two-stage game that the provider first selects the offloading service fee q, and then N users are the price-takers, selecting whether each of them subscribes to the offloading service or not.

A. Equilibrium Analysis

We state the main analytical result of this section in Theorem 3.1 as follows:

Theorem 3.1: For given p, b, N, and λ with user and provider rationality conditions, under sparse Wi-Fi regime (i.e., $\lambda d < 1$), there exists a unique equilibrium at which the offloading price $q^\star$, $N_I(q^\star)$, and $N_P(q^\star)$, are given by:

$$q^\star = p - \frac{b(1-e^{-\lambda d})}{2} \quad (9)$$

$$N_P(q^\star) = N\frac{b\lambda d}{2}, \qquad N_I(q^\star) = N(1 - \frac{b\lambda d}{2}), \quad (10)$$

Prior to presenting the proof, we first provide interpretations of the above theorem in what follows:

(a) Impact of parameters on offloading price: Two interpretations which naturally follow our intuitions are drawn in terms of b, λ, and d. First, in terms of the impact of the operation cost b, as it increases, the offloading price $q^\star$ decreases, and the user portion of selecting the offloading service increases. Second, as more Wi-Fi APs are installed or Wi-Fi APs are installed at more popular places (i.e., large λ), or the deadline d increases, the provider is willing to launch the offloading service with lower price and more users tend to select the offloading service.

(b) Provider's revenue: To see the effect of offloading service on the provider's revenue, we have the following from (4):

$$R(q^\star) = N\frac{b\lambda d}{2}\Big(p - \frac{b}{2}(1+e^{-\lambda d})\Big) + N(1-\frac{b\lambda d}{2})(p-b) \quad (11)$$

$$= N(p-b) + N\frac{b\lambda d}{2}\Big(\frac{b}{2}(1-e^{-\lambda d})\Big). \quad (12)$$

The second term of (12) corresponds to the additional revenue given to the provider due to offloading service. As in (11), this revenue increase can be understood as the cost reduction from b to $\frac{b}{2}(1+e^{-\lambda d})$, because until the deadline d, Wi-Fi is exploited for data delivery and the cellular cost b is only applied after the deadline d.

(c) Users' surplus: From users' perspective, the service fee reduction compensates for the disutility due to data delivery delay. When $q^\star = p - \frac{b(1-e^{-\lambda d})}{2}$, the expected user surplus (i.e., utility difference between being patient and impatient) is:

$$(1-p)(1-\frac{b\lambda d}{2}) + \int_0^{\frac{b\lambda d}{2}} \Big(\pi_\theta(P) - \pi_\theta(I)\Big)\mathrm{d}\theta,$$

which is always positive since $(1-p)(1-\frac{b\lambda d}{2}) + \frac{b^2}{8}\frac{1-e^{-\lambda d}}{\lambda d} > 0$.

We now present the proof of Theorem 3.1.

Proof of Theorem 3.1. The key to the proof lies in how users respond to the offloading price q chosen by the provider, based on which the provider will choose the optimal $q^\star$ that maximizes its revenue $R(q)$. As illustrated in Fig. 1, the utility of a user with smaller θ uniformly exceeds that of other users with larger delay sensitivity for all delivery times. Thus, we are first interested in the threshold $\bar{\theta}$, where users with $\theta \le \bar{\theta}$ (resp. $\theta > \bar{\theta}$) would choose to be patent (resp. impatient). Then, from (2) and (3), $\bar{\theta}$ should satisfy the following:

$$\pi_{\bar{\theta}}(P) - \pi_{\bar{\theta}}(I) \;=\; \bar{\theta}\frac{(e^{-\lambda d}-1)}{\lambda d} - q + p \;=\; 0, \quad (13)$$

where the above holds when $q = p - \frac{1-e^{-\lambda d}}{\lambda d}$. Then, we easily see that if $q < p - \frac{1-e^{-\lambda d}}{\lambda d}$, $\bar{\theta} \ge 1$, corresponding to when all users become patient, and if $q > p - \frac{1-e^{-\lambda d}}{\lambda d}$, users are split into patient and impatient ones. Also, recall that we have the provider rationality condition $be^{-\lambda d} < q < p$. Thus, to compute the equilibrium, it is convenient to divide into two cases: *(i)* $be^{-\lambda d} < p - \frac{1-e^{-\lambda d}}{\lambda d}$, and *(ii)* $p - \frac{1-e^{-\lambda d}}{\lambda d} \le be^{-\lambda d}$.

case (i): In this case, as mentioned earlier, depending on q, we have different users' response. When $q \le p - \frac{1-e^{-\lambda d}}{\lambda d}$, $\bar{\theta} \ge 1$, and all users become patient, and thus, $R(q)$ is simply a linearly increasing function of q, given by:

$$R(q) = (q - be^{-\lambda d})N, \quad (14)$$

which is maximized when $q = p - \frac{1-e^{-\lambda d}}{\lambda d}$. However, when $p - \frac{1-e^{-\lambda d}}{\lambda d} < q < p$, we have $\bar{\theta} < 1$, and thus we have both patient and impatient users whose portions are $\bar{\theta}$ and $1-\bar{\theta}$. From (4) and (13), it is easy to see that $R(q)$ is quadratic and we can easily prove that $q^\star$ maximizing $R(q)$ is given by:

$$q^\star \;=\; p - \frac{b(1-e^{-\lambda d})}{2}. \quad (15)$$

Also, we can check $R(q^\star)$ for $q^\star$ in (15) exceeds the revenue obtained by (14) when $q = p - \frac{1-e^{-\lambda d}}{\lambda d}$.

case (ii): In this case, we should have $q > p - \frac{1-e^{-\lambda d}}{\lambda d}$, because of provider rationality condition. Thus, some users become patient and other users become impatient, and similarly to the earlier case, $R(q)$ is quadratic in q, maximized by $q^\star$ in (15) This completes the proof. ■.

IV. Analysis: Market with Two Providers

A. Equilibrium Analysis

We first present the main analysis result on the equilibrium of this market in what follows:

Theorem 4.1: For given p, b, k, λ and switching cost c with user and provider rationality conditions, under sparse Wi-Fi regime (i.e., $\lambda d < 1$), there exists a unique equilibrium at which the provider A's offloading price $q^\star$ is given by:

$$q^\star = \begin{cases} \frac{1}{2}p + \frac{1}{2}be^{-\lambda d} + \frac{k(p-b)-c}{2(k+1)} & \text{if } 0 \le c < \bar{c}, \\ p - \frac{b(1-e^{-\lambda d})}{2} & \text{if } \bar{c} \le c < p - be^{-\lambda d}, \end{cases} \quad (16)$$

for some threshold switching cost $\bar{c}$ (which will be specified in the proof). Also, at the equilibrium, we have the following number of user shares:

(i) $0 \le c < \bar{c}$:

$$N_{P_A}(q^\star) = \gamma N_A, \quad N_{I_A}(q^\star) = (1-\gamma)N_A,$$
$$N'_{P_A}(q^\star) = (\gamma - \frac{\lambda dc}{1-e^{-\lambda d}})N_B,$$
$$N_{I_B}(q^\star) = (1 - \gamma + \frac{\lambda dc}{1-e^{-\lambda d}})N_B,$$

where
$$\gamma = \frac{\lambda d}{1-e^{-\lambda d}}\left(\frac{1}{2}(p - be^{-\lambda d}) - \frac{k(p-b)}{2(k+1)} + \frac{c}{2(k+1)}\right).$$

(ii) $\bar{c} < c < p - be^{-\lambda d}$:

$$N_{P_A}(q^\star) = N_A\frac{b\lambda d}{2}, \qquad N'_I(q^\star) = N_A(1 - \frac{b\lambda d}{2})$$
$$N'_{P_A}(q^\star) = 0, \qquad N'_I(q^\star) = N_B.$$

B. Interpretation of Theorem 4.1

(a) Impact of parameters on offloading price. When the switching cost c is sufficiently small, the provider A attracts the provider B users to use the offloading service, whereas the market is separated otherwise. We call such regimes *subscription change* and *no subscription change* regimes, respectively, where $\bar{c}$ turns out to be the threshold switching cost. According to the characterization of the offloading price $q^\star$ in the subscription change regime, $q^\star$ decreases as switching cost c increases, and as Wi-Fi meeting rate λ or deadline d increases, the provider are willing to launch the offloading service with lower price and more provider B users tend to subscribe the offloading service from A. In addition, $q^\star$ is related to the original user share, quantified by k, where recall that if $k > 1$, the provider A had larger user share. Thus, larger k permits the provider A to set a higher $q^\star$.

(b) User share. Now, how does the user share change after the offloading service is launched? First, obviously, in the no subscription change regime (i.e., the market is separated), there is no change in the user share. However, in the subscription change regime for low switching cost, a certain portion of provider B users subscribes the offloading service. To quantify it, i.e., $N'_{P_A}(q^\star)$, from Theorem 4.1, we get:

$$N'_{P_A}(q^\star) = \frac{\lambda d}{1-e^{-\lambda d}}\Big(\frac{1}{2}(p - be^{-\lambda d}) - \frac{k}{2(k+1)}(p-b) - (1 - \frac{1}{2(k+1)})c)\Big)N_B.$$

As the inter Wi-Fi AP contact rate λ increases or deadline d increases, more provider B users become patient with provider A's offloading service. However, as the switching cost c increases, less provider B users change its subscription, because $1 - \frac{1}{2(1+k)} > 0$.

C. Proof of Theorem 4.1

Given the offloading price q of the provider A, users in both providers respond differently. We first divide this market into two regimes: ***R1.*** $q > p - c$ and ***R2.*** $q \le p - c$, where ***R1*** corresponds to the case when the provider B users do not use offloading service from A, since switching cost is two high, whereas in ***R2***, there exists some portion of provider B users having low delay sensitivity, who subscribes to the provider A's offloading service. It is clear that in ***R1***, the market is separated, so the analysis is similar to that of the monopoly market. Provider A will choose the optimal price $q^\star$ that maximizes its revenue by comparing the revenues in both regimes.

<u>*Step 1.*</u> Thus, as a first step, we analysis how the provider A selects the optimal offloading price $q^\star$ in what follows:

*(i) Regime **R1** ($p - c \le q < p$):* As discussed earlier, this case is similar to that of the monopoly market, as described in Theorem 3.1, where the revenue maximizing $q^\star_{R1}$ in this regime is given by:

$$q^\star_{R1} = \begin{cases} p - \frac{b(1-e^{-\lambda d})}{2} & \text{if } p - \frac{b(1-e^{-\lambda d})}{2} > p - c \\ p - c & \text{otherwise,} \end{cases} \quad (17)$$

where note that if $p - \frac{b(1-e^{-\lambda d})}{2} < p - c$, the revenue is monotonically decreasing in q over the interval $[p-c, p]$, so the maximum is achieved at $q = p - c$. Let

$$q'_{R1} = p - \frac{b(1-e^{-\lambda d})}{2}.$$

Then, from (7) and using the result (12) from the monopoly market, the provider A's revenue is expressed by:

$$R(q) = N_{I_A}(q)(p-b) + (N_{P_A}(q) + N'_{P_A}(q))(q - be^{-\lambda d}), \quad (18)$$

*(ii) Regime **R2** ($be^{-\lambda d} < q < p - c$):* In this case, as done in the analysis of the monopoly market, to compute the provider A's revenue, it is crucial to find out the delay sensitivity thresholds in both providers A and B users. Let us denote such thresholds θ_A and θ_B, respectively. Since those thresholds are computed by comparing the net-utilities between being patient and impatient, it is not hard to see that θ_A and θ_B are computed by solving the following equations:

$$\pi_{\theta_A}(I_A) = \pi_{\theta_A}(P_A), \quad \pi_{\theta_B}(I_B) = \pi'_{\theta_B}(P_A),$$

where recall that $\pi'_{\theta_B}(P_A)$ is the (expected) net-utility when a provider B user chooses to use the offloading service offered by the provider A, as stated in (6). Solving the above, we have θ_A and θ_B as follows:

$$\theta_A = \frac{p-q}{\frac{1-e^{-\lambda d}}{\lambda d}}, \qquad \theta_B = \frac{p-(q+c)}{\frac{1-e^{-\lambda d}}{\lambda d}}. \quad (19)$$

Note that from (7) the provider A's revenue in this case is expressed by:[3].

$$R_{R2}(q) = (p-b)N_A(1-\theta_A) + (q - be^{-\lambda d})(N_A\theta_A + N_B\theta_B). \quad (20)$$

Then, it is easy to see that the above $R(q)$ is quadratic in q, having the maximum, achieved by $q = q'_{R1}$:

$$q'_{R2} = \frac{1}{2}p + \frac{1}{2}be^{-\lambda d} + \frac{k(p-b)-c}{2(k+1)}$$

Then, the revenue maximizing $q^\star_{R2}$ is given by:

$$q^\star_{R2} = \begin{cases} q'_{R2} & \text{if } be^{-\lambda d} < q'_{R2} < p-c \\ p-c & \text{if } q'_{R2} \geq p-c, \end{cases} \quad (21)$$

where when $q'_{R2} > p-c$, $R(q)$ is monotonically increasing, so the maximum is achieved at $q = p - c$.

From (17), (18), (20), and (21), the provider A chooses the optimal $q^\star$ as follows:

$$q^\star = \begin{cases} q^\star_{R1} & \text{if} \quad R_{R1}(q^\star_{R1}) \geq R_{R2}(q^\star_{R2}) \\ q^\star_{R2} & \text{otherwise.} \end{cases} \quad (22)$$

Step 2. As a second step, we show that the structure of $q^\star$ can be simplified by showing that $q^\star$ cannot be $p-c$, i.e.,

$$q^\star = \begin{cases} q'_{R1} & \text{if} \quad R_{R1}(q'_{R1}) \geq R_{R2}(q'_{R2}) \\ q'_{R2} & \text{otherwise.} \end{cases} \quad (23)$$

The above means that the optimal offloading price is always either q'_{R1} or q'_{R2}. The possibility of $q^\star = p-c$ comes from the following three cases, each of which will be shown not to occur or to imply (23).

a) $q^\star_{R1} = p-c$ and $q^\star_{R2} = q'_{R2}$: Since $q'_{R2} \in [be^{-\lambda d}, p-c)$ where $q^\star_{R2} = q'_{R2}$ and $R_{R1}(p-c) = R_{R2}(p-c)$, we should have $q^\star = q'_{R2}$.

b) $q^\star_{R1} = q'_{R1}$ and $q^\star_{R2} = p-c$: Similarly to *a)*, since $q^\star_{R1} \in (p-c, p]$, where $q^\star_{R1} = q'_{R1}$ and $R_{R1}(p-c) = R_{R2}(p-c)$, we should have $q^\star = q'_{R1}$.

c) $q^\star_{R1} = p-c$ and $q^\star_{R2} = p-c$. From (17), $q^\star_{R1} = p-c$, implying $q'_{R1} < p-c$. Similarly, from (21), $q^\star_{R2} = p-c$, implying $q'_{R2} \geq p-c$. This is a contradiction, because $q'_{R1} = p - \frac{b(1-e^{-\lambda d})}{2} > q'_{R2} = \frac{1}{2}p + \frac{1}{2}be^{-\lambda d} + \frac{k(p-b)-c}{2(k+1)}$.

Step 3. By regarding $R_{R1}(\cdot)$ and $R_{R2}(\cdot)$ as the functions of switching cost c, let us denote $\bar{c}$ be c which satisfies:

$$R_{R1}(q'_{R1}) = R_{R2}(q'_{R2}). \quad (24)$$

Since the function $R_{R2}(\cdot)$ is (strictly) decreasing in c (which can be easily shown by checking $\frac{\mathrm{d}R_{R2}(q'_{R2})}{\mathrm{d}c} < 0$), we have:

$$\begin{cases} R_{R2}(q'_{R2}) > R_{R1}(q'_{R1}) & \text{if} \quad 0 \leq c < \bar{c} \\ R_{R1}(q'_{R1}) \leq R_{R2}(q'_{R2}) & \text{otherwise.} \end{cases} \quad (25)$$

Then, from (23) and (25), the result follows. ■

[3]We consider $\theta_A < 1$, $\theta_B < 1$ where $be^{-\lambda d} > p - \frac{1-e^{-\lambda d}}{\lambda d}$. This means that the minimum price for offloading service is larger than the price where all user can use offloading service. This is reasonable, because MNO's major business seems focused on the cellular service and offloading service is additional service.

V. Numerical Analysis

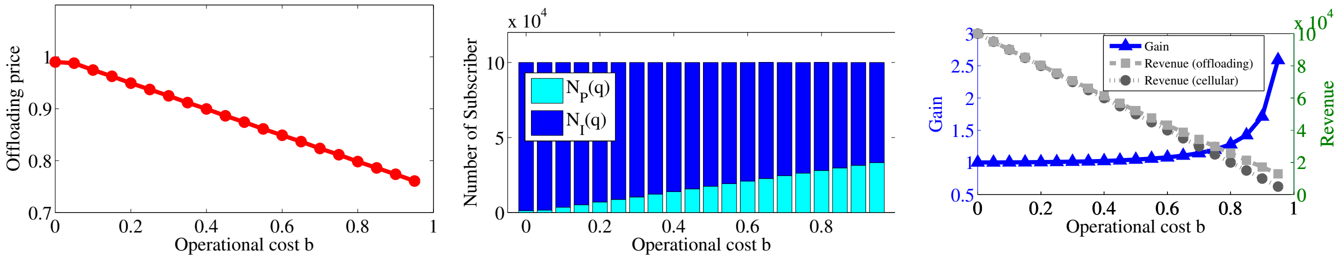
This section shows our numerical analysis for the monopoly market and the market with two providers. For the monopoly case, we analyze how the offloading price, the number of subscribers and the revenue change according to the operational cost and Wi-Fi meeting rate. For the two providers in the market, we analyze how the offloading price, the number of subscribers and the revenue of two providers change according to the switching cost and the original user shares (prior to introducing offloading service).

In all numerical analysis, we used the cellular operation cost $b = 0.7$ and the price of a cellular service $p = 1$ and the number of subscribers of both provdiers $N_A = N_B = 100000$. To choose deadline d and Wi-Fi meeting rate λ, we extracted the parameter values from the measurement data set in [4] which were 93 iPhone users' Wi-Fi connectivity data captured at every 3 minutes for two weeks in South Korea. The measurement results in [4] tells us that the average Wi-Fi contact probability in residential areas for 10 min (600 sec) deadline is 0.7, so we set $d = 600$ (sec) and $\lambda = 0.0012$.

A. Monopoly Market

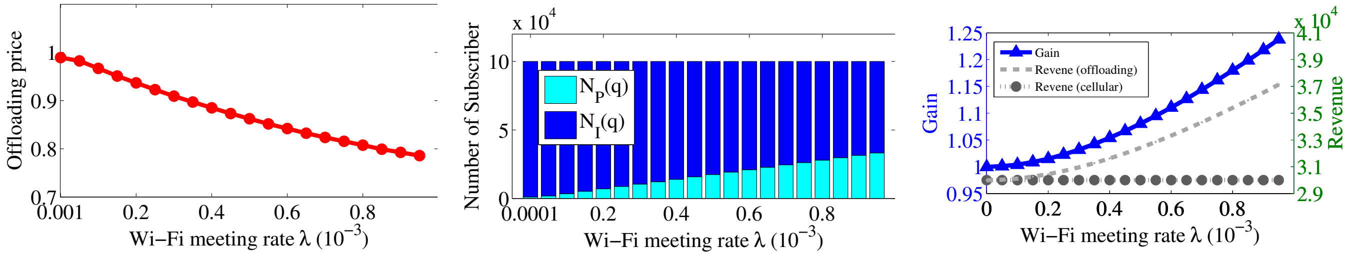
***(a) Impact of operational cost for a cellular service* b:** It is expected that the Wi-Fi offloading service is effective when the operational cost of a cellular service is expensive. To verify this expectation, we investigate how the operational cost impact on the offloading price of a provider, the number of users choosing the offloading service, and the revenue. Fig. 3(a) depicts that the offloading price decreases as the operational cost, b increases. This implies that the provider reduces the cellular traffic by giving users high incentive to encourage to using offloading service when an operational cost for a cellular service is high. Fig. 3(b) shows that the number of users choosing offloading service, N_{P_A}, increases as the operational cost increases. This implies that many users choose offloading service rather than cellular-only service in response to the incentive the provider gives to them. We examine the ratio of the revenue with offloading service to that without offloading service (a cellular-only service), which is called "gain" in Fig. 3(c). The higher the gain, the more effective the offloading service. Fig. 3(c) shows that the gain is increasing as the operational cost grows, as expected. Note that the revenues decrease for both offloading service and cellular-only service when the operational cost for a cellular service increases.

***(b) Impact of Wi-Fi meeting rate* λ:** Note that λ depends on the number of installed Wi-Fi APs; the more the number of Wi-Fi APs, the higher λ; more Wi-Fi APs imply lead to lower. Fig 4(a) depicts that the offloading price decreases as the number of Wi-Fi APs installed increases. The reason is that many users subscribe to offloading service if many Wi-Fi APs are deployed, because of small probability of disconnection from Wi-Fi or small delay (waiting time) for Wi-Fi, which results in high utility of offloading service. Thus, as λ increases, the offloading service becomes more attractive to users, thus, the number of patient users $N_{P_A}(q^\star)$ increases (i.e., the decrease in the number of impatient users $N_{I_A}(q^\star)$) as shown in Fig. 4(b). Fig 4(c) shows the revenue



(a) Offloading price (b) Number of subscriber (c) Revenue

Fig. 3. Impact of the service cost for unit cellular traffic, b



(a) Offloading price (b) Number of subscriber (c) Provider's revenue

Fig. 4. Impact of the inter Wi-Fi AP contact rate, λ.

with offloading service, the revenue with cellular-only service (before introducing offloading service), and the gain (the ratio of the former to the latter). As the Wi-Fi meeting rate increases, the gain increases up to about 1.25 times.

B. Market with Two Providers

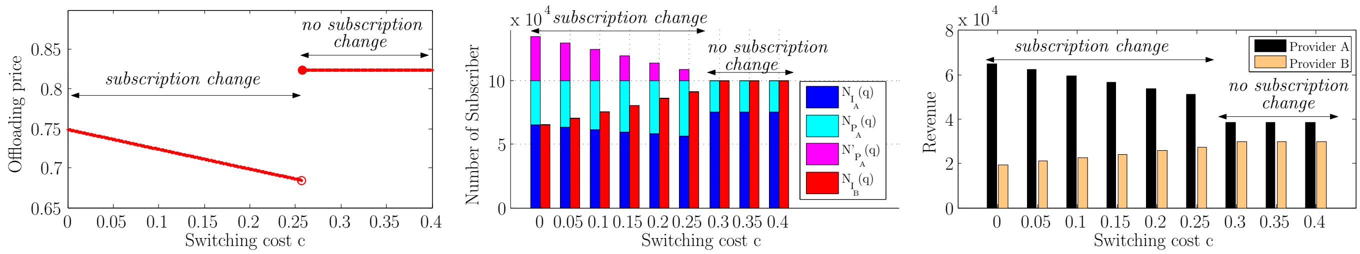
This subsection numerically analyzes the case that one provider (provider A) offers offloading service and another provider (provider B) offers only cellular service. We assume that a user pays a switching cost c when he switches his subscription from one provider to another provider in using the offloading service. Our interests are 1) whether subscribers of the provider B change their decision to use offloading service and 2) how such decision impacts on the offloading price, the market share and the revenue of both providers.

(a) Impact of switching cost: The offloading price shows different behaviors with respect to various c as in Fig 5(a). There is a threshold value in the switching cost c such that below this threshold, where the switching cost c is small, provider A strategically attracts some subscribers of provider B and above this threshold, where c is sufficiently large, provider A attracts none of provider B's subscribers.

Fig. 5(b) describes this in more detail. Recall that $N_{I_A}(q^\star)$ $(N_{I_B}(q^\star))$ is the number of impatient users of provider A (B, respectively) and $N_{P_A}(q^\star)$ the number of patient users of provider A. Note that $N'_{P_A}(q^\star)$ is the number of provider B's patient users who switches from provider A to provider B to use offloading service. In Fig. 5(b), the threshold value of c is 0.26. That is, when $c < 0.26$, some of provider B users switch from provider B to provider A to subscribe to the offloading service which is offered only by provider A. When $c \geq 0.26$, none of provider B's subscribers changes his decision. Based on the above result, we call that when $c < 0.26$, both providers are in the *subscription change* regime and when $c \geq 0.26$, both providers are in the *no subscription change* regime.

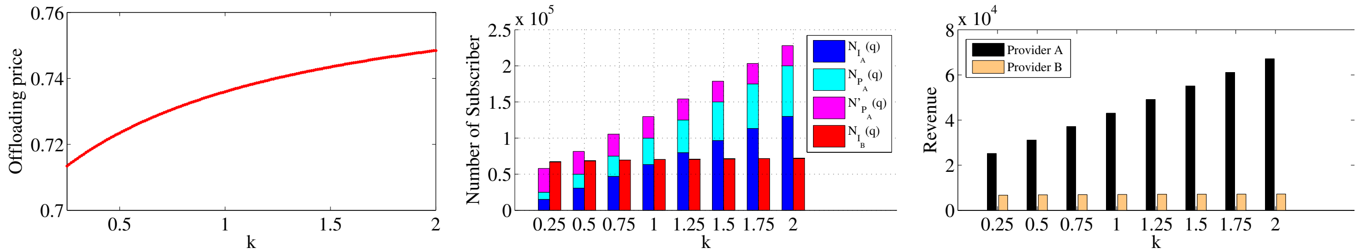
In the *subscription change* regime, the offloading price decreases as c increases. This can be explained as follows. When the switching cost is low, provider A attracts subscribers of provider B by reducing the offloading service to increase the number subscribers of offloading service (especially the users of provider B). Hence for high switching cost, provider A gives high incentive to attract provider B's customer as long as $c < 0.26$. However, reducing offloading service in fact causes the revenue decrease as in Fig. 5(c). Once $c \geq 0.26$, no subscriber of provider B switches his decision. Therefore, for $c > 0.26$, if provider A reduces offloading service price, it will experience revenue decreasing without attracting a user of provider B. Hence provider A stops reducing its offloading service price at $c = 0.26$. Since we assume that $N_A = N_B$, we can regard the revenue of provider A as the revenue of the provider in the monopoly case with offloading service and the revenue of provider B as that of the provider without offloading service in the monopoly case. According to the results for the monopoly case in the previous subsection, providing offloading service has the bigger revenue than only cellular service. This explains why the revenue of provider A is greater than that of provider B where $c > 0.26$ in Fig. 5(c).

(b) Impact of original user share k under subscription change regime: Recall that if $k > 1$, the provider A has larger user share than provider B where $N_A = kN_B$. Fig 6(b) shows the user shares with varying k. As k increases, the portion of provider B' users who use offloading service decreases. Since the original user share of provider A is sufficiently larger than that of provider B, the provider A increases its offloading service fee to maximize its revenue rather than decreases to



(a) Offloading price (b) Number of subscriber (c) Providers' revenue

Fig. 5. Impact of switching cost $c = [0, 0.4]$.



(a) Offloading price (b) Number of subscriber (c) Providers' revenue

Fig. 6. Impact of original user share, k where $c = 0.05$ with *subscription change* regime.

attract the provider B's users to use offloading service (see 6(a)). Thus, the more original user share the provider A has, the less provider B's users use offloading service (see $N'_{P_A}(q^\star)$ in Fig 6(b)). For example, when $k = 0.25$, about 43% of provider B's users use offloading service and when $k = 2$, about 12% of provider B's users use offloading service.

VI. Conclusion and Future Work

In this paper, we use a game theoretic framework to study the economic aspects of the delayed Wi-Fi offloading service in monopoly market as well as a market with two providers where only one provider (provider A) launches the offloading service and some users of another provider (provider B) can subscribe to provider A's offloading service with switching cost. We drew the following messages from analytical and numerical studies: (a) Offloading service is beneficial to both providers and users. (b) As the operational cost for a cellular service increases, more Wi-Fi APs are installed, or longer deadline that a user can set for delaying its delay-tolerant data, the provider is willing to launch a offloading service with lower price to increase its revenue. (c) In the market with two providers, when the switching cost is smaller than a certain threshold, the provider A chooses lower offloading price to attract the provider B users to use offloading service, whereas, when the switching cost exceeds than the threshold, the provider A does not attract provider B users and sets its offloading price as the same offloading price as in monopoly market. As a future work, our analysis can be extended in multi-provider markets to study competition effects where each provider simultaneously launches offloading service.

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